Implicit-Explicit Time-Integration Methods

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1 Overview of the Field

Many physical systems are modeled by partial differential equations that consist of contributions from distinct physical processes, e.g., advection, diffusion, or chemical reactions. These contributions typically have different numerical properties. It is therefore a natural idea to employ different methods for their numerical solution [1, 5, 7, 3]. The subject area of this Research in Teams workshop is the study of implicit-explicit time-integration methods, which employ a divide-and-conquer strategy to treat stiff and non-stiff parts of a differential equation and can lead to significant improvements in computational efficiency [6, 2, 4].

2 Recent Developments and Open Problems

Although there is a considerable body of research accumulated over the past three decades, only very simple IMEX time integration methods are being employed in applications that need them most. We believe this is primarily due to a lack of accessibility of this body of research by the community at large.

The success of IMEX methods boils down to the tripartite tradeoff between stability of the method, the accuracy of the solution, and the performance of the implementation (which includes scalability to massively parallel computing architectures). The implementation of IMEX methods often involves the existence of specialized methods for certain types of physics, but it also depends crucially on how to effectively split the problem in the first place.

Accordingly, the main open problems in the design of new and more effective IMEX methods can be broadly grouped into three categories:

- balance stability, accuracy, and performance,
- a lack of a suitable test problem for linear stability,
- lack of a comprehensive test suite of representative problems.

3 Presentation Highlights

This workshop was part of the Research in Teams program and hence did not include any formal presentations. Any presentations made were informal and spontaneous in nature and were essentially either review of existing theory or software or summarizing actual progress made in these areas during the workshop.

4 Scientific Progress Made

Scientific progress was made in the following areas:

- theory for linear stability for IMEX problems,
- general-purpose software for IMEX methods,
- creation of a comprehensive test suite of IMEX problems,
- written material for the monograph.

5 Outcome of the Workshop

For the past five years, the applicants (Spiteri and Sandu) have been working on a research monograph that will largely disseminate the advances in the field of implicit-explicit time-integration methods. This monograph is under contract with SIAM. At the time of the workshop, the applicants were in the final stages of completing a first draft of the monograph. The main objective of this Research in Teams workshop is to provide the applicants with dedicated and undistracted time to wrap up these final stages. The monograph would be the first of its kind and represents a timely contribution to the explosion in interest and application of IMEX time-integration methods reflected in the literature.

In addition to the scientific progress made as listed in the previous section, the overall outcome of the workshop was a fruitful and inspiring time and renewed energy toward completing the monograph.

References

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