



irphé

Institut de Recherche sur les
Phénomènes Hors Equilibre

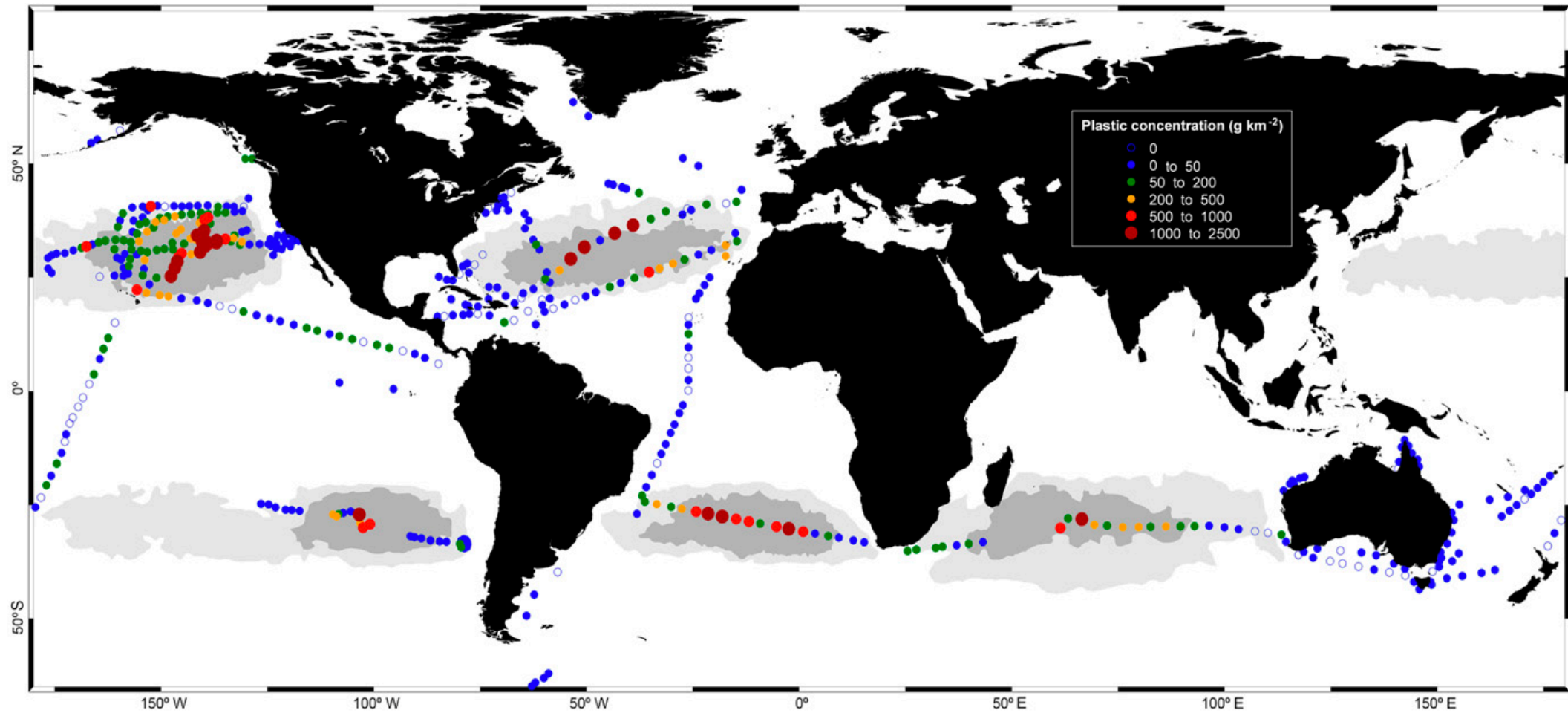


Modeling the deformation and the fragmentation of brittle objects in turbulence

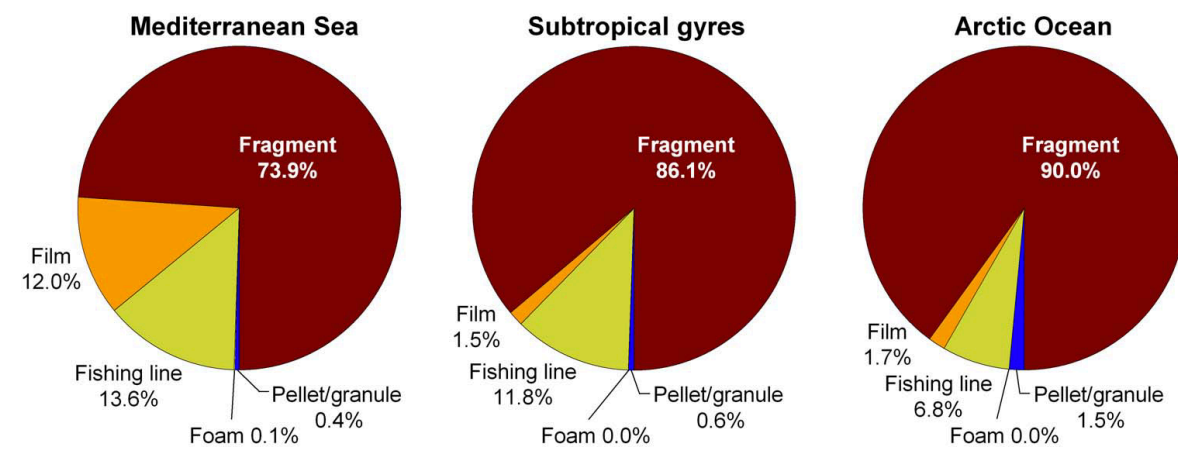
Gautier Verhille

C. Brouzet, M.J. Dalbe, B. Favier, A. Gay, P. Le Gal, N. Vandenberghe, E. Villermaux

Motivation

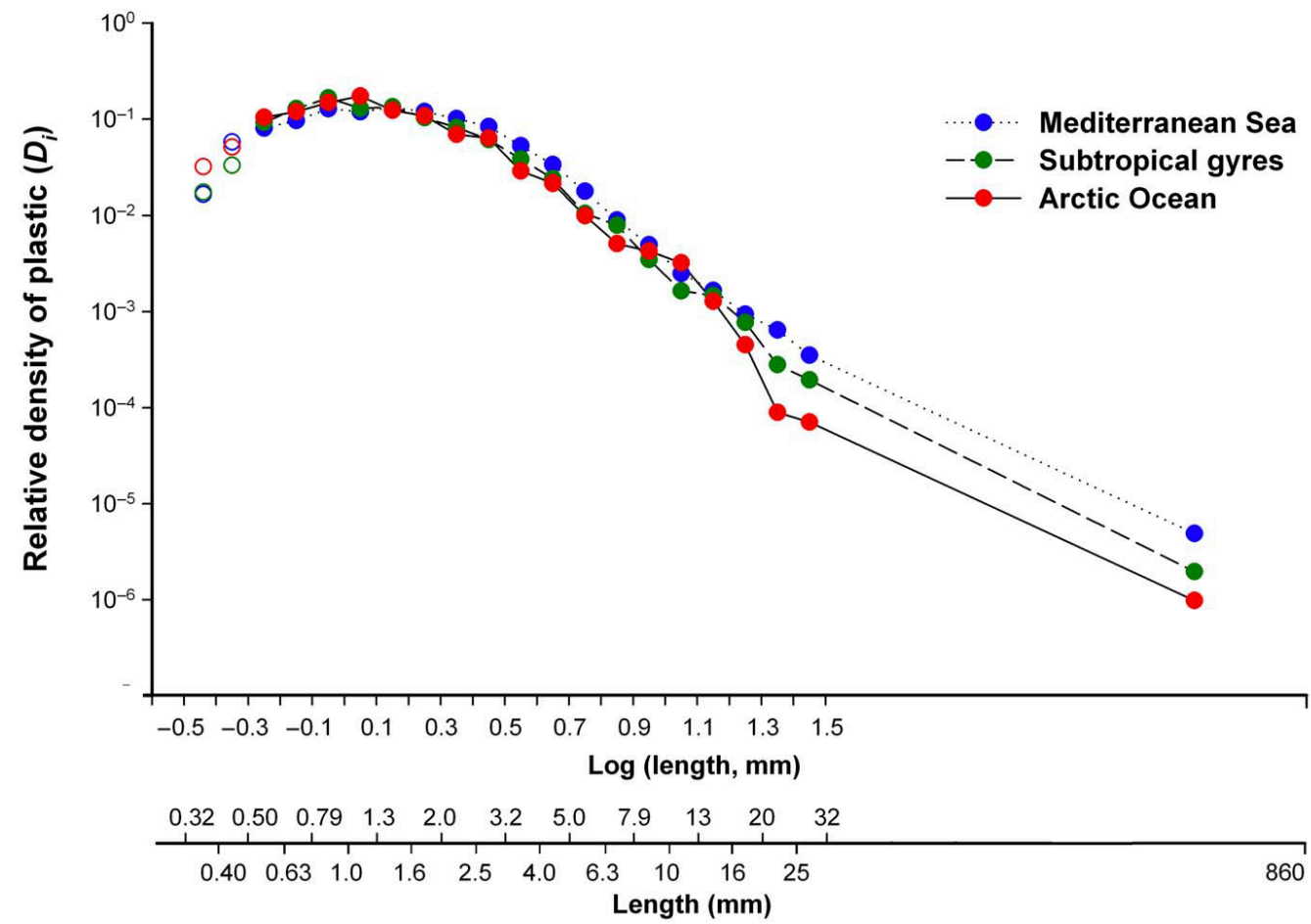


Cozar *et al.*, PNAS, 2014

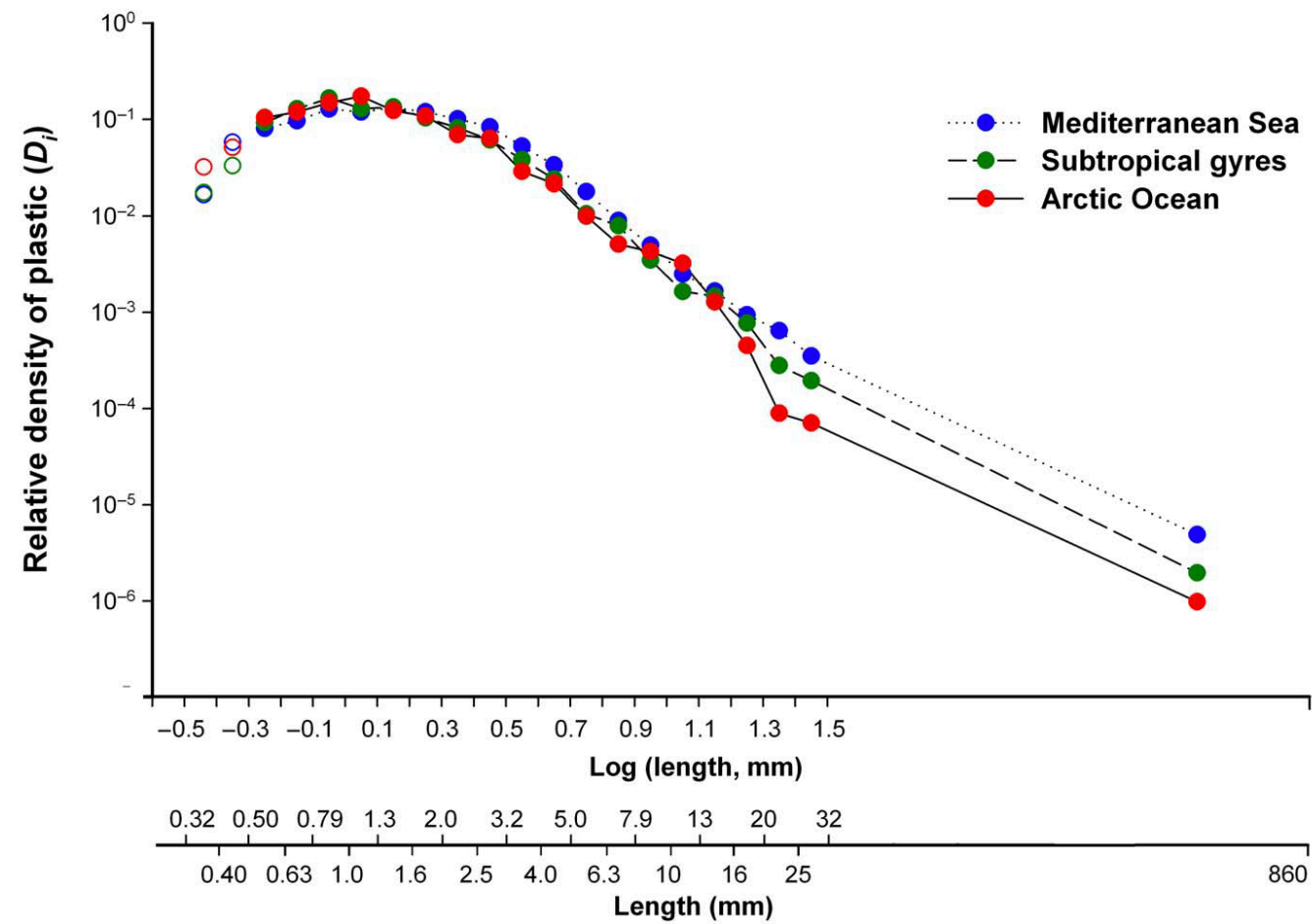


Cozar *et al.*, Science Advances, 2017

Motivation



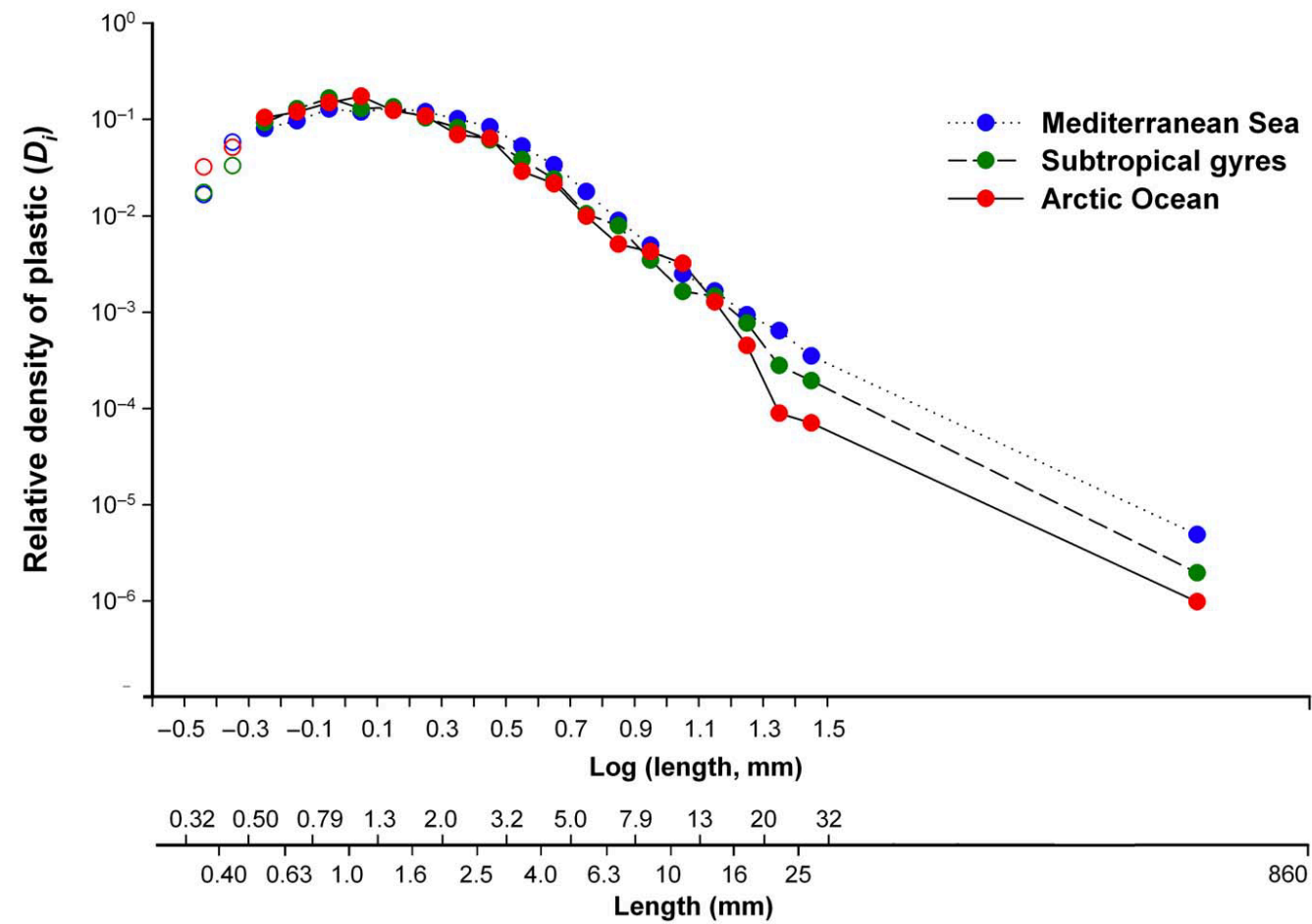
Motivation



Size distribution independent of the localization

Is there a generic mechanism?

Motivation

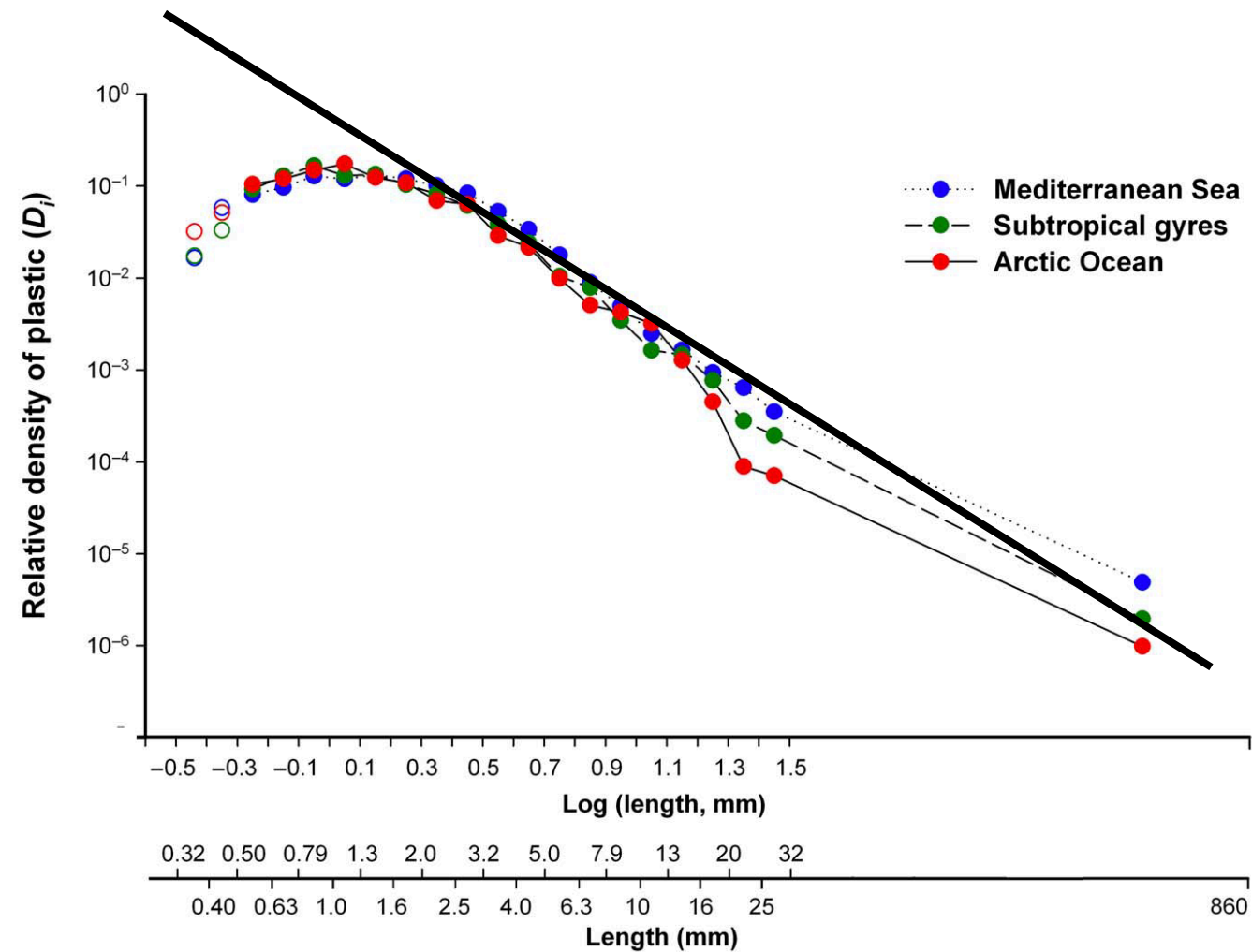


Size distribution independent of the localization

Is there a generic mechanism?

Two step mechanisms: embrittlement/fragmentation

Motivation



Size distribution independent of the localization

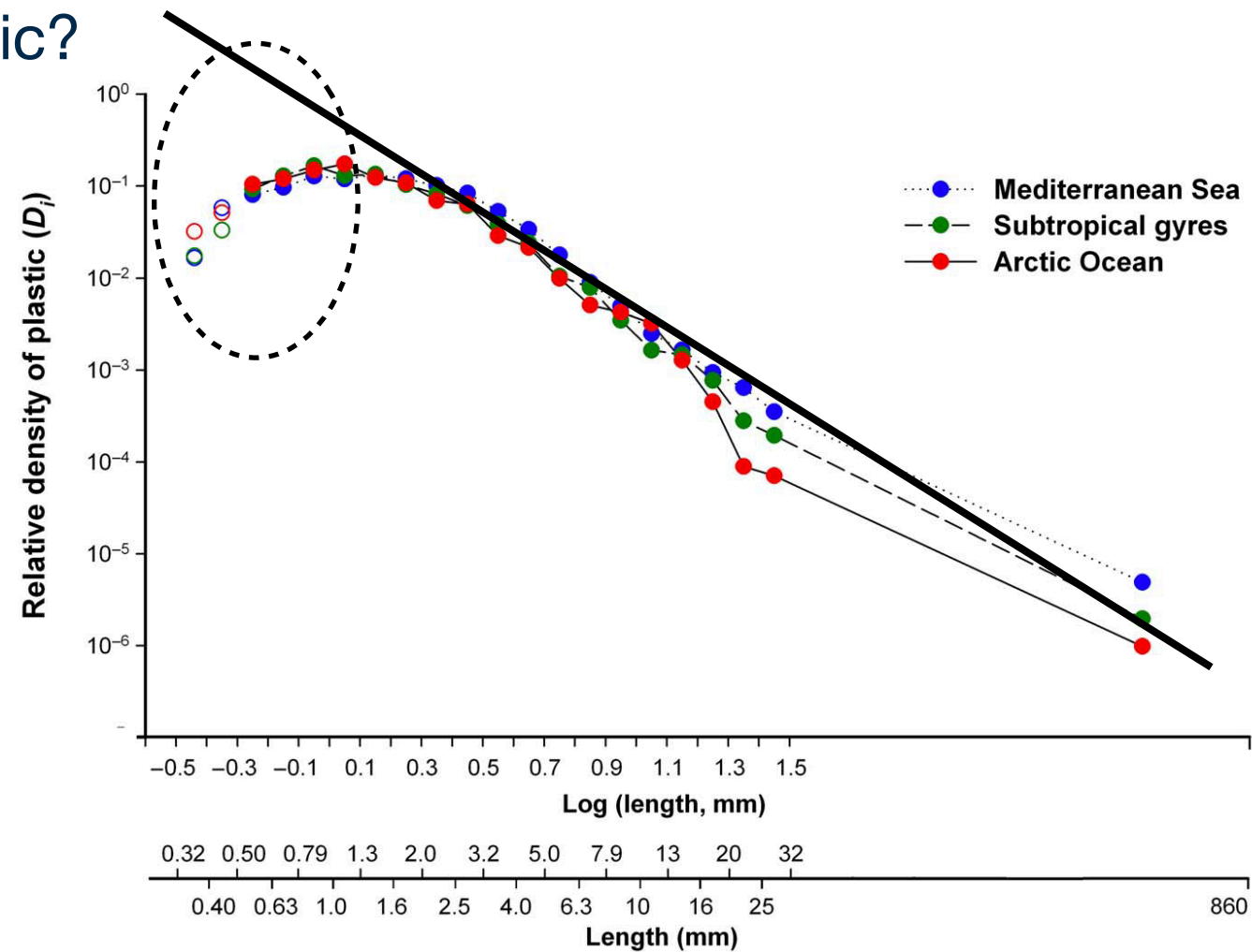
Is there a generic mechanism?

Two step mechanisms: embrittlement/fragmentation

Model for fragmentation proposed by Cozar et al: power law

Motivation

missing plastic?



Size distribution independent of the localization

Is there a generic mechanism?

Two step mechanisms: embrittlement/fragmentation

Model for fragmentation proposed by Cozar et al: power law

Motivation

Cozar's model based on

PRL **104**, 095502 (2010)

PHYSICAL REVIEW LETTERS

week ending
5 MARCH 2010

New Universality Class for the Fragmentation of Plastic Materials

G. Timár,^{1,2} J. Blömer,³ F. Kun,¹ and H. J. Herrmann^{2,4}

¹*Department of Theoretical Physics, University of Debrecen, P. O. Box:5, H-4010 Debrecen, Hungary*

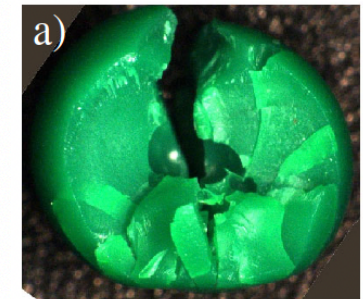
²*Computational Physics IfB, HIF, ETH, Hönggerberg, 8093 Zürich, Switzerland*

³*Spezialwerkstoffe, Fraunhofer Institute UMSICHT, Osterfelder Strasse 3, 46047 Oberhausen, Germany*

⁴*Departamento de Física, Universidade Federal do Ceara, 60451-970 Fortaleza, Ceara, Brazil*

(Received 21 December 2009; published 3 March 2010)

We present an experimental and theoretical study of the fragmentation of polymeric materials by impacting polypropylene particles of spherical shape against a hard wall. Experiments reveal a power law mass distribution of fragments with an exponent close to 1.2, which is significantly different from the known exponents of three-dimensional bulk materials. A 3D discrete element model is introduced which reproduces both the large permanent deformation of the polymer during impact and the novel value of the mass distribution exponent. We demonstrate that the dominance of shear in the crack formation and the plastic response of the material are the key features which give rise to the emergence of the novel universality class of fragmentation phenomena.



DOI: [10.1103/PhysRevLett.104.095502](https://doi.org/10.1103/PhysRevLett.104.095502)

PACS numbers: 62.20.M-, 46.50.+a, 64.60.-i

Mechanism very unlikely in the ocean

Our assumption: plastic breaks due to large deformation during storm

Outline

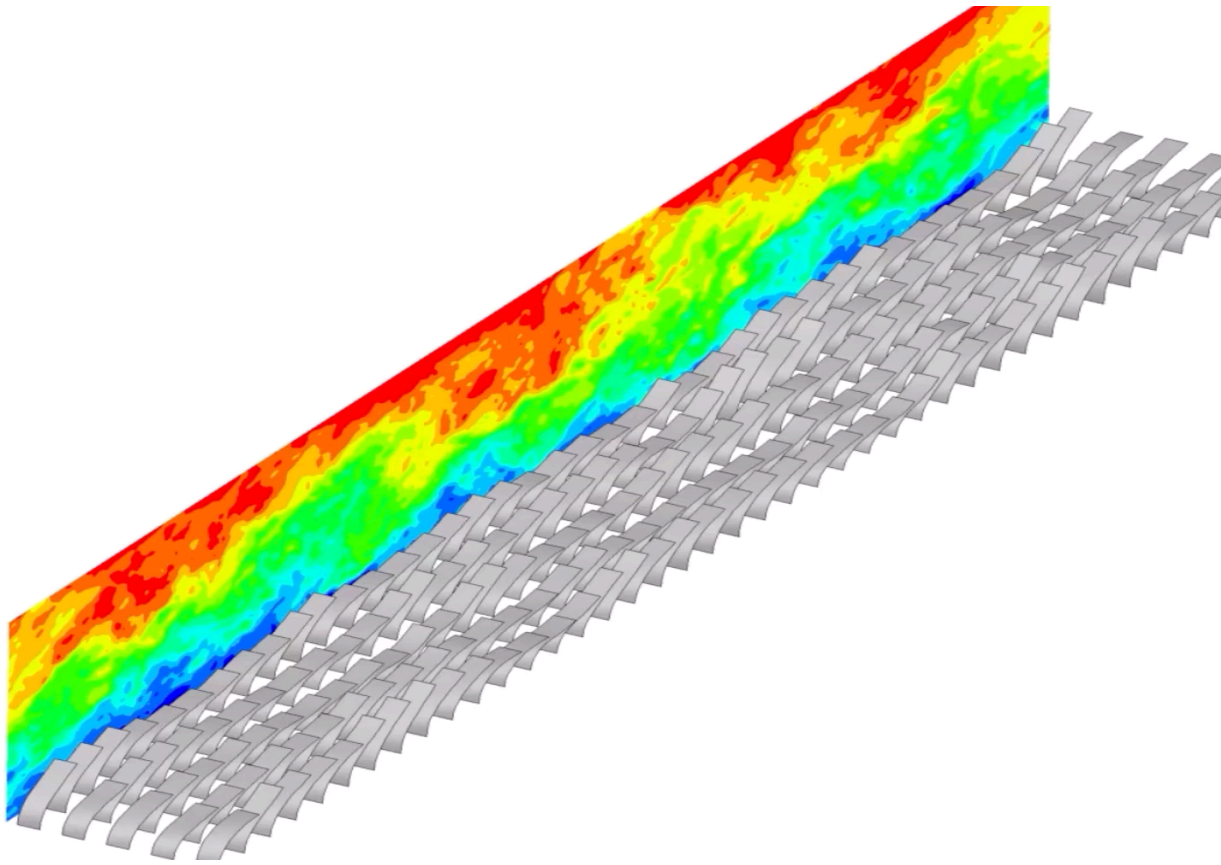
- Particle deformation in turbulence
- Fragmentation: deformation of brittle particles

Particle deformation

Particle deformation

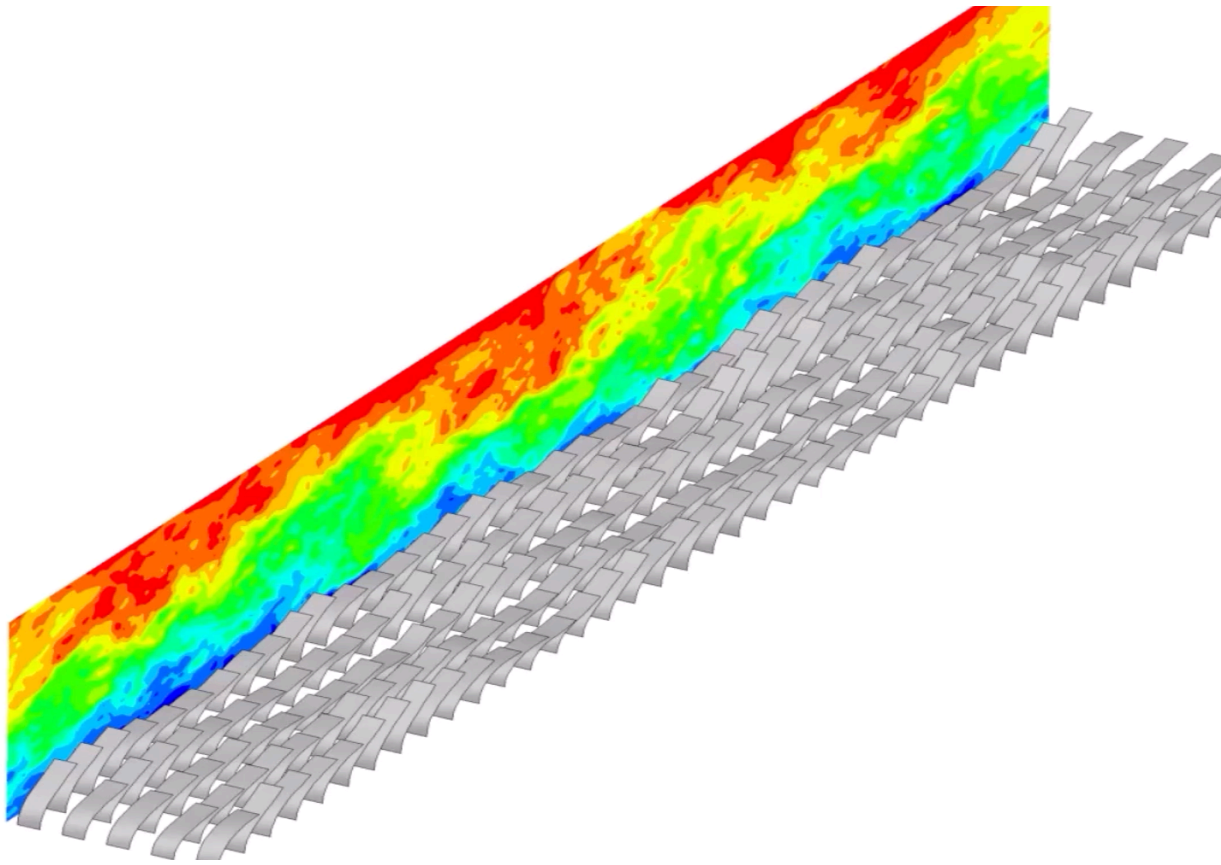
- When does a particle deform in a turbulent flow?
- How does a particle deform in a turbulent flow?

Classical fluid-structure approach



Univ. Minnesota

Classical fluid-structure approach



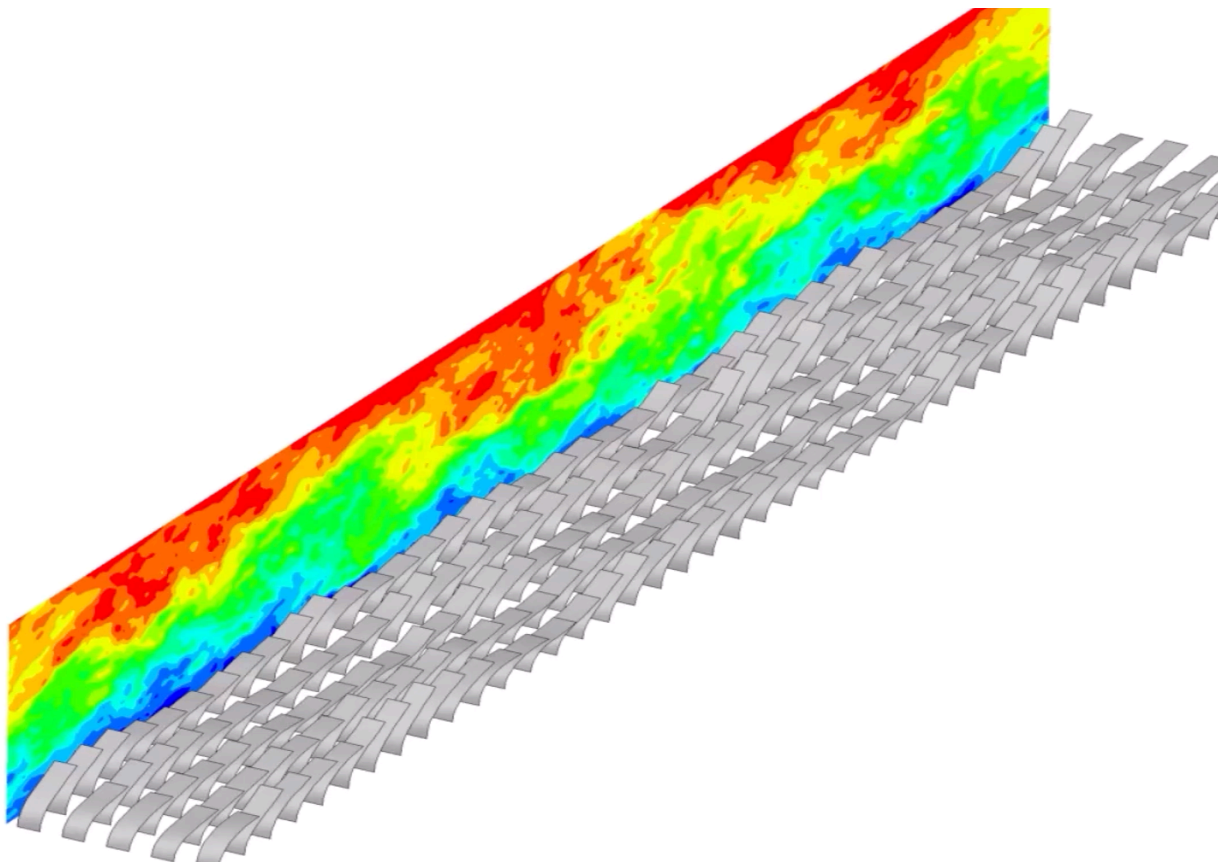
Univ. Minnesota

Classical fluid-structure approach

Turbulent flow

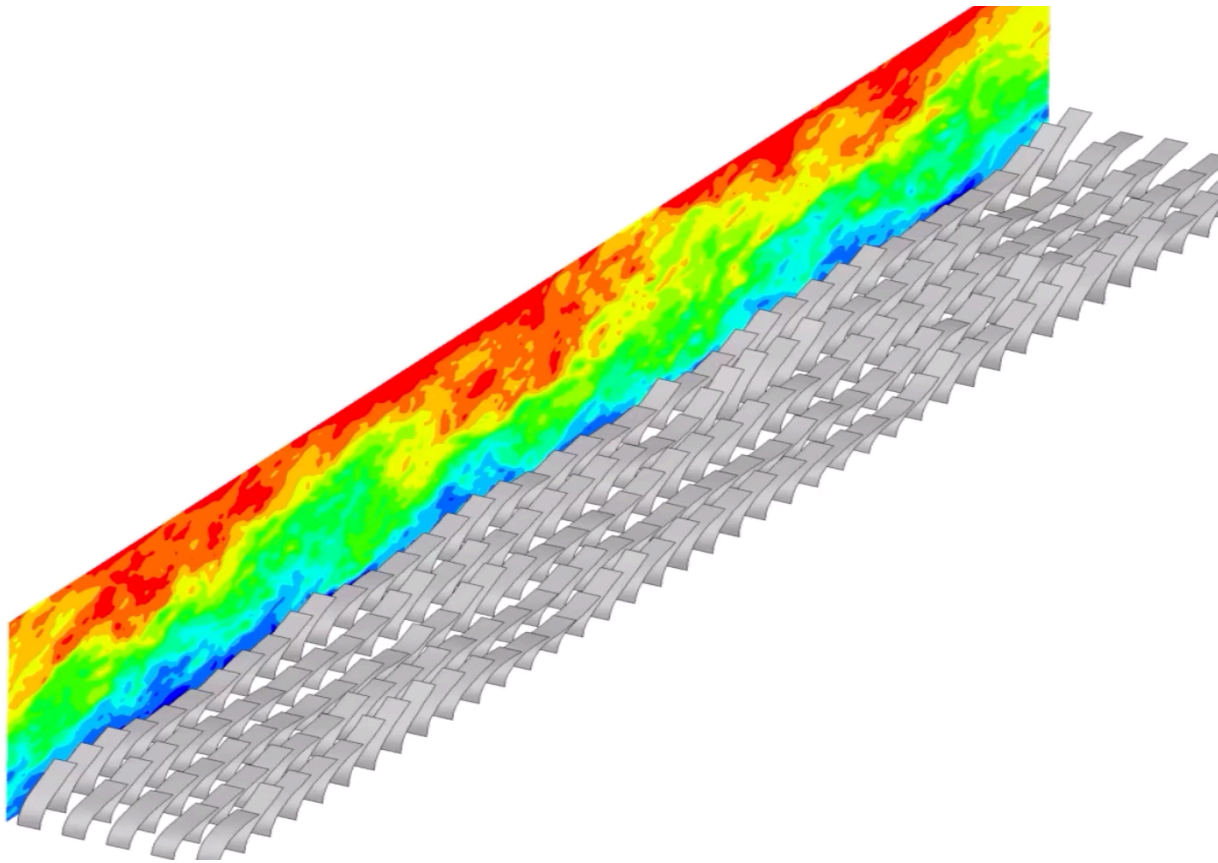
Pressure force \sim Bending force

$$\rho S U^2 \sim \frac{EI}{L^2}$$



Univ. Minnesota

Classical fluid-structure approach



Univ. Minnesota

Turbulent flow

Pressure force \sim Bending force

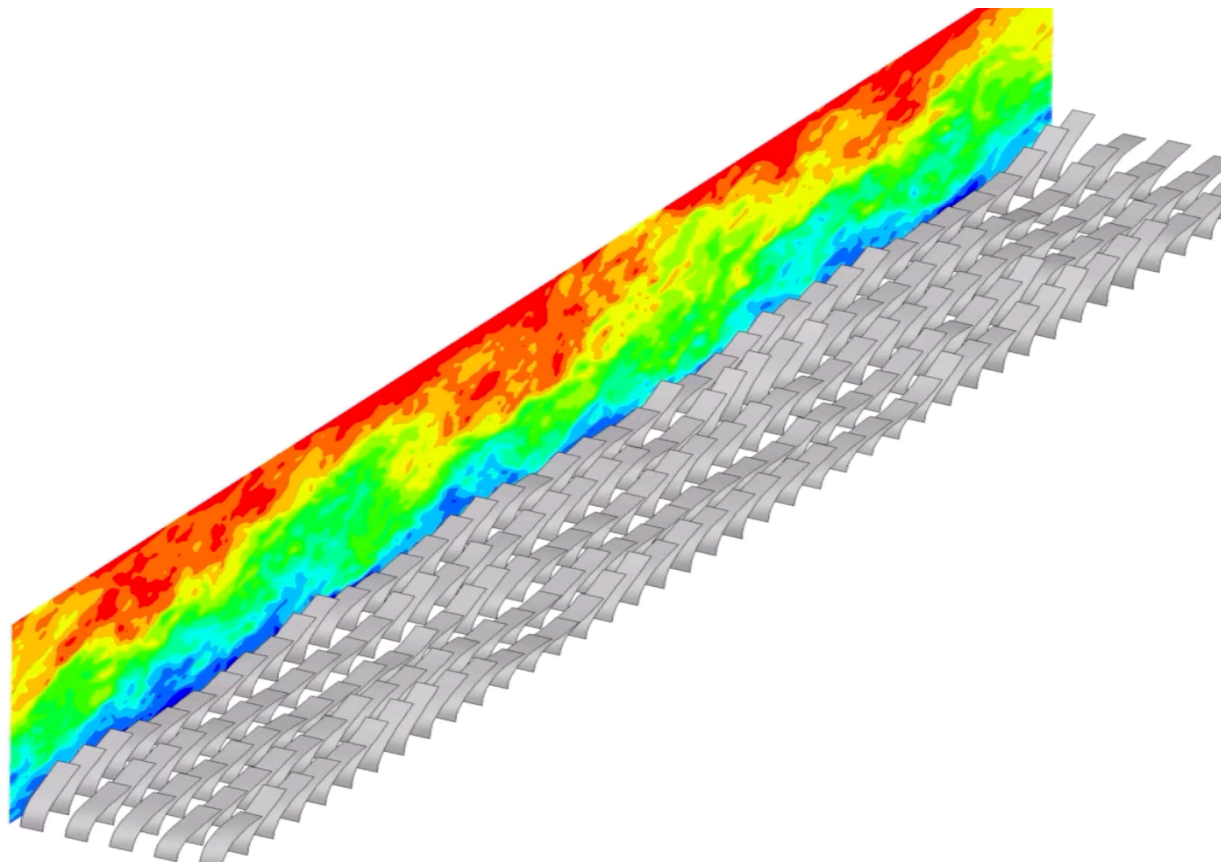
$$\rho S U^2 \sim \frac{EI}{L^2}$$

Laminar flow

Viscous force \sim Bending force

$$\eta L U \sim \frac{EI}{L^2}$$

Classical fluid-structure approach



Univ. Minnesota

Turbulent flow

Pressure force \sim Bending force

$$\rho S U^2 \sim \frac{EI}{L^2}$$

Laminar flow

Viscous force \sim Bending force

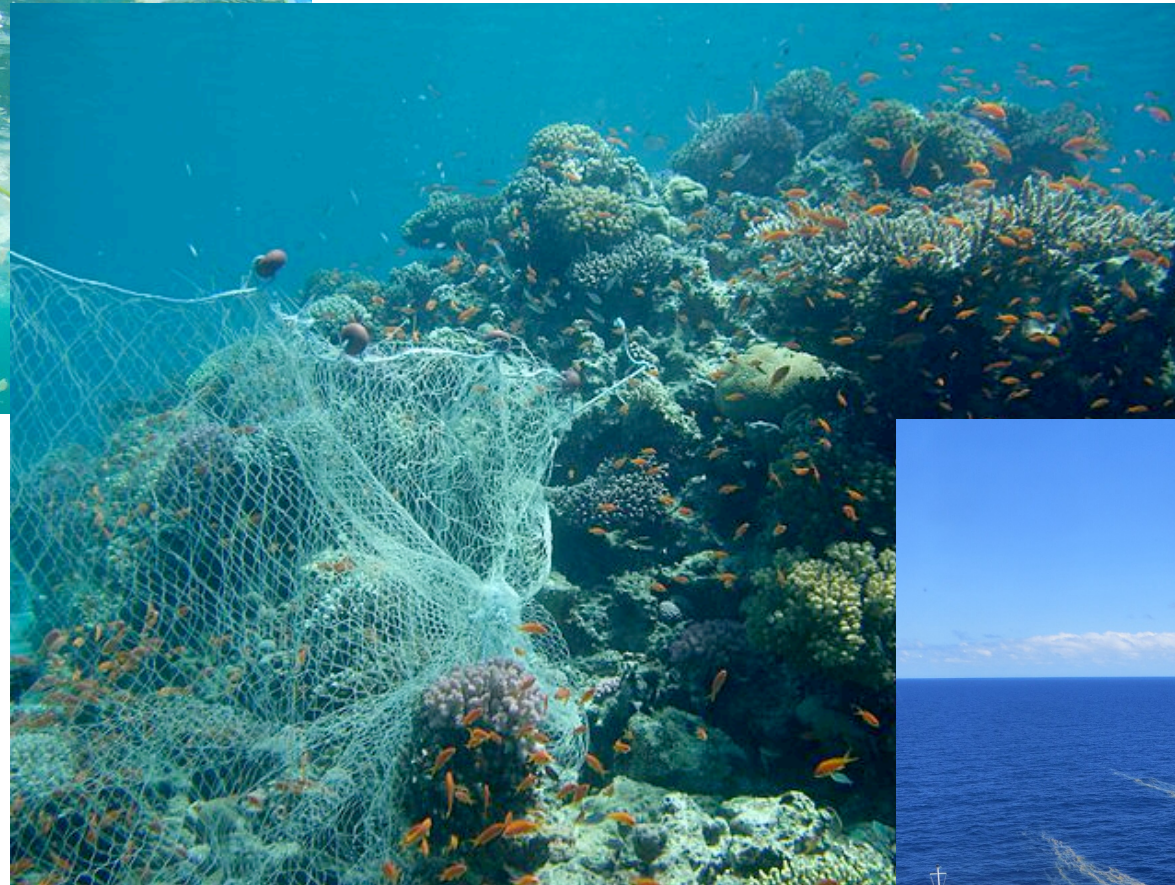
$$\eta L U \sim \frac{EI}{L^2}$$

Here pressure/viscous force also responsible of particle advection.
Which length scale matters for the velocity?

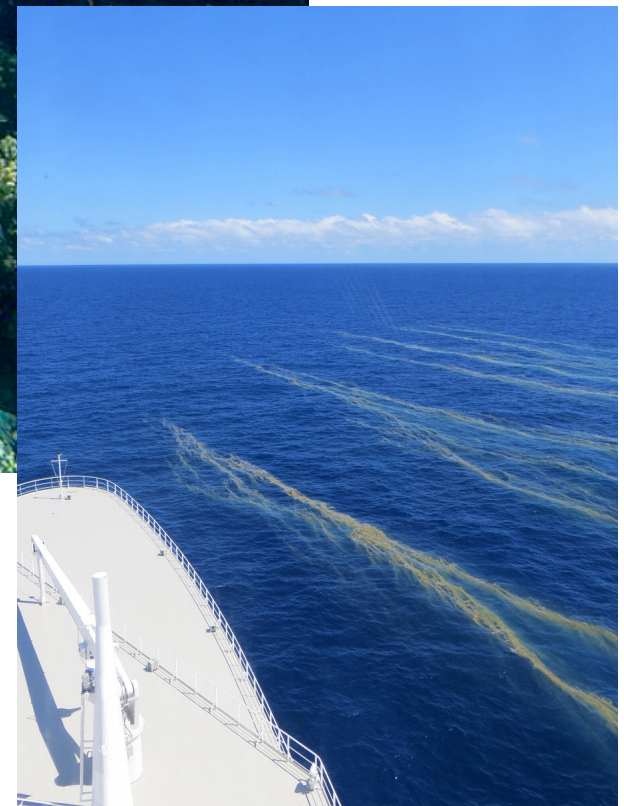
Various shapes in the ocean



Plastic bag, bottle, ...



Fishing line, net, ...

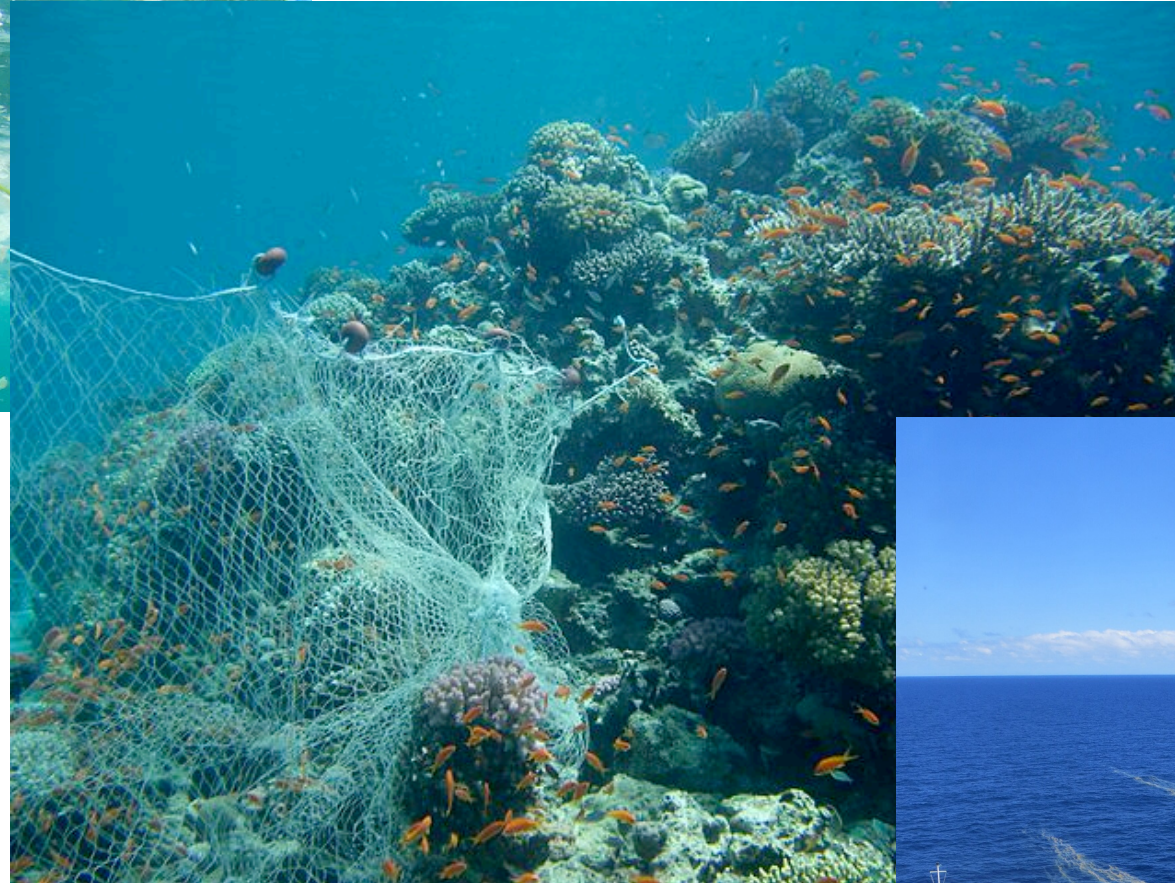


plankton colony
(Trichodesmium)

Various shapes in the ocean



Plastic bag, bottle, ...



Fishing line, net, ...



plankton colony
(Trichodesmium)

Here simplest shapes:
Fibers (mainly)
Discs

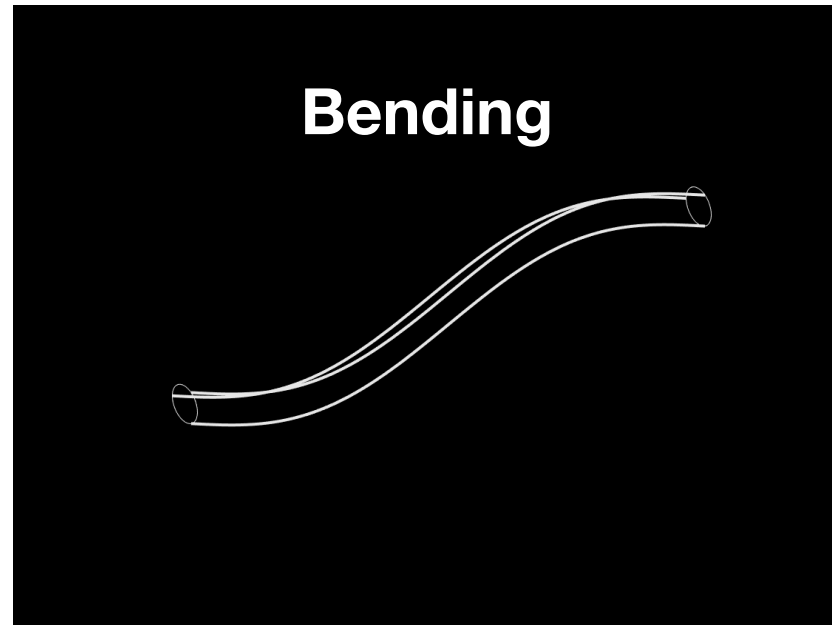
The different mode of deformation

$$\sigma \partial_{tt} \mathbf{r} - \partial_s (T \partial_s \mathbf{r}) + EI \partial_s^4 \mathbf{r} = \boldsymbol{\xi} \quad , \quad |\partial_s \mathbf{r}|^2 = 1 \quad , \quad \boldsymbol{\xi} = \eta (\mathbf{u} - \partial_t \mathbf{r})$$

The different mode of deformation

$$\sigma \partial_{tt} \mathbf{r} - \partial_s (T \partial_s \mathbf{r}) + EI \partial_s^4 \mathbf{r} = \boldsymbol{\xi} \quad , \quad |\partial_s \mathbf{r}|^2 = 1 \quad , \quad \boldsymbol{\xi} = \eta (\mathbf{u} - \partial_t \mathbf{r})$$

Timescale of the deformation



$$EI \partial_s^4 \mathbf{r} \sim \eta \partial_t \mathbf{r}$$

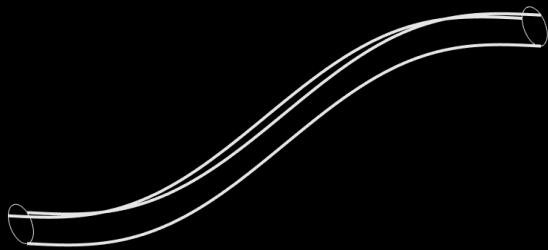
$$\tau_b \sim \frac{\eta L^4}{EI}$$

The different mode of deformation

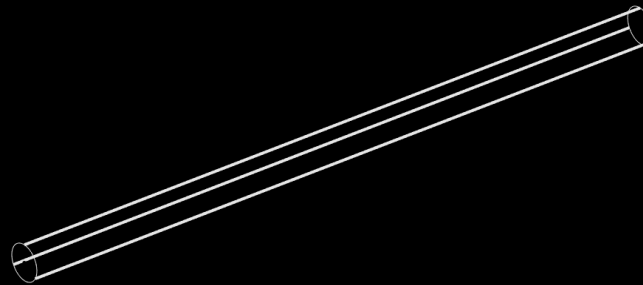
$$\sigma \partial_{tt} \mathbf{r} - \partial_s (T \partial_s \mathbf{r}) + EI \partial_s^4 \mathbf{r} = \boldsymbol{\xi} \quad , \quad |\partial_s \mathbf{r}|^2 = 1 \quad , \quad \boldsymbol{\xi} = \eta (\mathbf{u} - \partial_t \mathbf{r})$$

Timescale of the deformation

Bending



Stretching



$$EI \partial_s^4 \mathbf{r} \sim \eta \partial_t \mathbf{r}$$

$$\tau_b \sim \frac{\eta L^4}{EI}$$

$$Ed^2 \partial_{ss} \mathbf{r} \sim \eta \partial_t \mathbf{r}$$

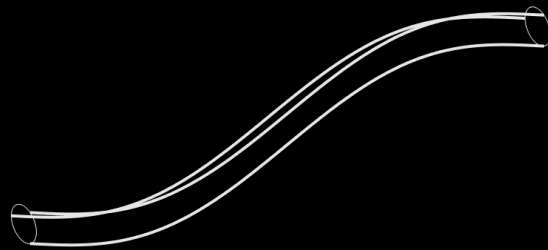
$$\tau_s \sim \frac{\eta L^2}{Ed^2}$$

The different mode of deformation

$$\sigma \partial_{tt} \mathbf{r} - \partial_s (T \partial_s \mathbf{r}) + EI \partial_s^4 \mathbf{r} = \boldsymbol{\xi} \quad , \quad |\partial_s \mathbf{r}|^2 = 1 \quad , \quad \boldsymbol{\xi} = \eta (\mathbf{u} - \partial_t \mathbf{r})$$

Timescale of the deformation

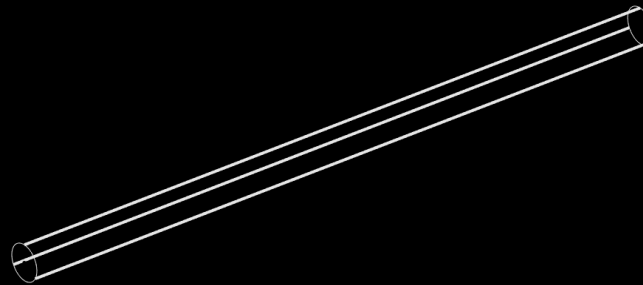
Bending



$$EI \partial_s^4 \mathbf{r} \sim \eta \partial_t \mathbf{r}$$

$$\tau_b \sim \frac{\eta L^4}{EI}$$

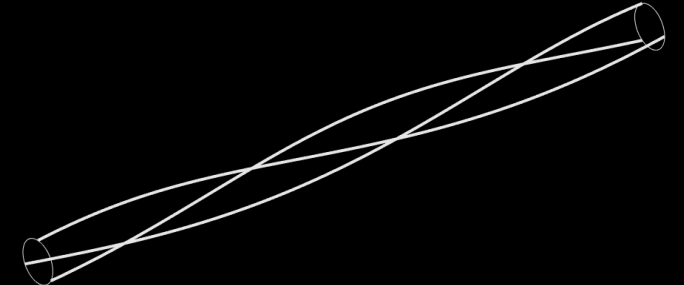
Stretching



$$Ed^2 \partial_{ss} \mathbf{r} \sim \eta \partial_t \mathbf{r}$$

$$\tau_s \sim \frac{\eta L^2}{Ed^2}$$

Twisting



$$C \partial_{ss} \theta \sim \eta d \partial_t d \theta$$

$$\tau_t \sim \frac{\eta d^2 L^2}{C}$$

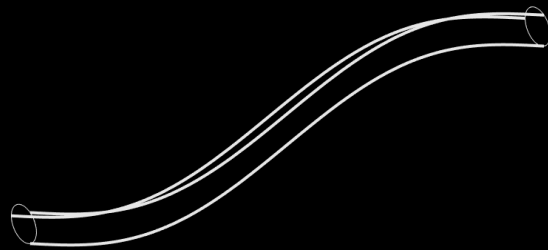
$$C \sim EI$$

The different mode of deformation

$$\sigma \partial_{tt} \mathbf{r} - \partial_s (T \partial_s \mathbf{r}) + EI \partial_s^4 \mathbf{r} = \boldsymbol{\xi} \quad , \quad |\partial_s \mathbf{r}|^2 = 1 \quad , \quad \boldsymbol{\xi} = \eta (\mathbf{u} - \partial_t \mathbf{r})$$

Timescale of the deformation

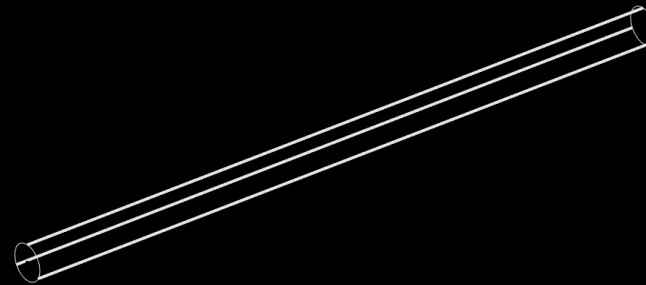
Bending



$$EI \partial_s^4 \mathbf{r} \sim \eta \partial_t \mathbf{r}$$

$$\tau_b \sim \frac{\eta L^4}{EI}$$

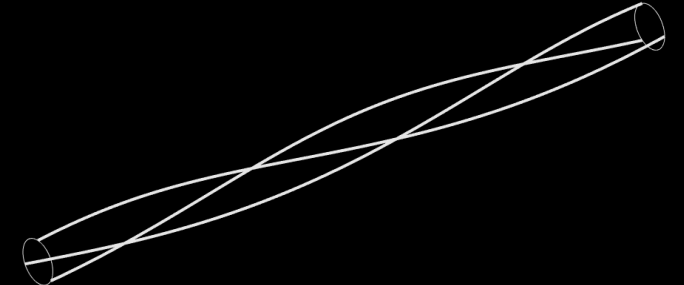
Stretching



$$Ed^2 \partial_{ss} \mathbf{r} \sim \eta \partial_t \mathbf{r}$$

$$\tau_s \sim \frac{\eta L^2}{Ed^2}$$

Twisting



$$C \partial_{ss} \theta \sim \eta d \partial_t d \theta$$

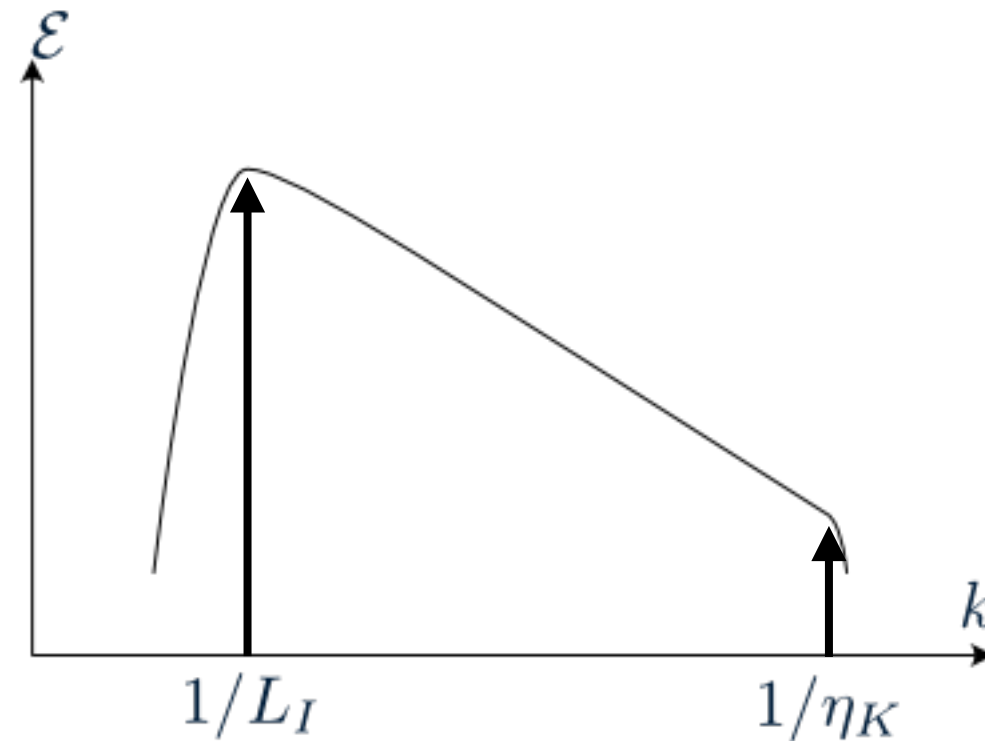
$$\tau_t \sim \frac{\eta d^2 L^2}{C}$$

$$C \sim EI$$

$$\text{For fibers } L \gg d \quad \tau_s \sim \tau_t \ll \tau_b$$

At first order fibers are inextensible and untwistable

Particle size vs turbulence length scales



$$\eta_K = \left(\frac{\nu^3}{\varepsilon} \right)^{1/4} \quad \tau_K = \left(\frac{\nu}{\varepsilon} \right)^{1/2}$$

- Particle smaller than Kolmogorov length
- Particle in the inertial regime
- Particle larger than the integral length

$$\sigma \partial_{tt} \mathbf{r} - \partial_s (T \partial_s \mathbf{r}) + EI \partial_s^4 \mathbf{r} = \boldsymbol{\xi} \quad , \quad |\partial_s \mathbf{r}|^2 = 1 \quad , \quad \boldsymbol{\xi} = \eta (\mathbf{u} - \partial_t \mathbf{r})$$

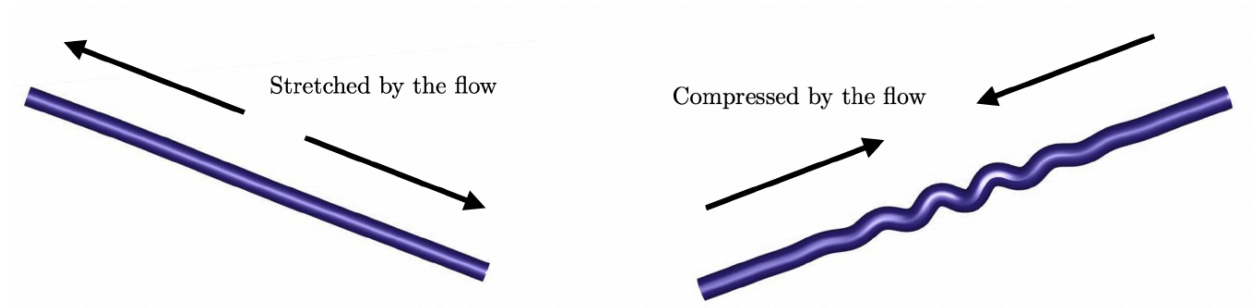
Different correlation for the forcing $\boldsymbol{\xi}$

Particle smaller than the Kolmogorov length

Smooth flow (at the scale of the fiber)

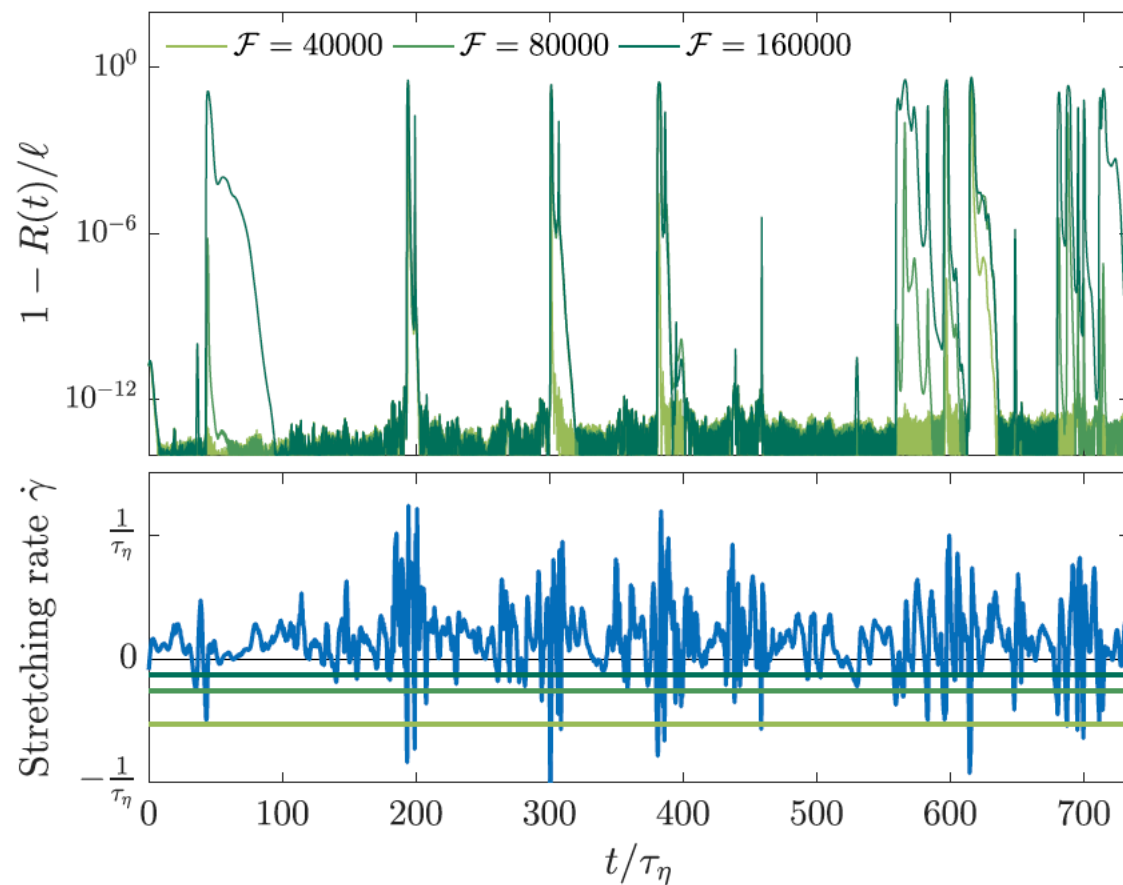
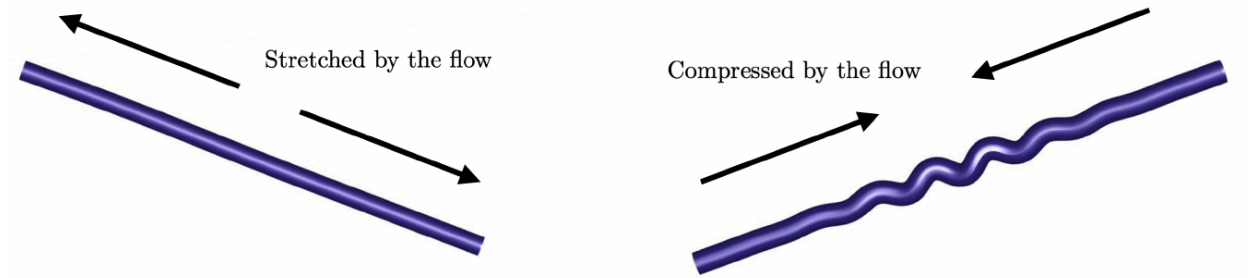
Particle smaller than the Kolmogorov length

Smooth flow (at the scale of the fiber)



Particle smaller than the Kolmogorov length

Smooth flow (at the scale of the fiber)



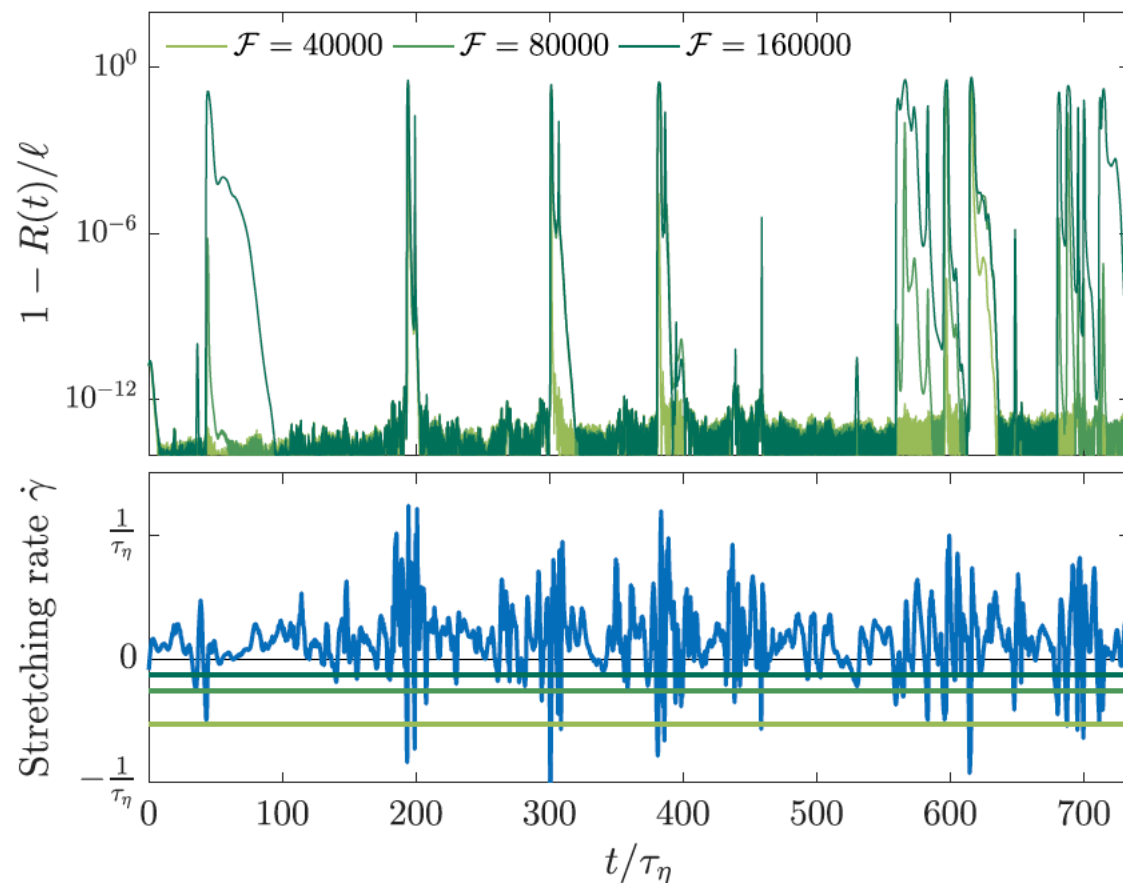
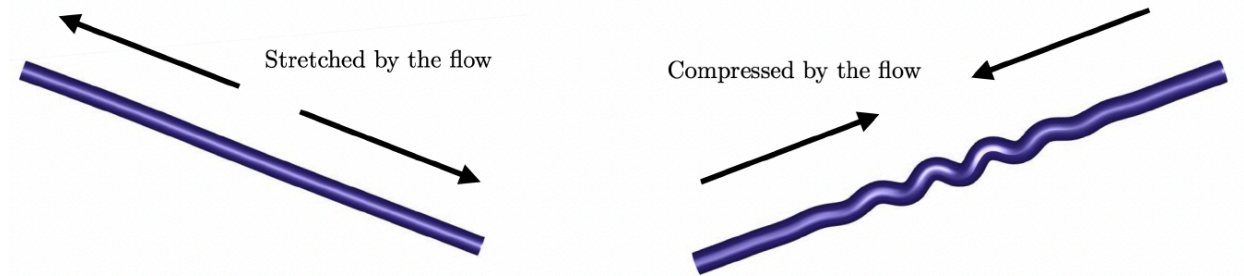
Flexibility parameter

$$\mathcal{F} = \frac{8\pi\rho_f\nu\ell^4}{cE\tau_\eta}$$

Viscous force \sim Bending force

Particle smaller than the Kolmogorov length

Smooth flow (at the scale of the fiber)



Strong intermittency of buckling event

Preferential alignment with the strain

Buckling if $\dot{\gamma}\tau_{\eta}\mathcal{F} < -\mathcal{F}^*$

Flexibility parameter

$$\mathcal{F} = \frac{8\pi\rho_f\nu\ell^4}{cE\tau_{\eta}}$$

Viscous force \sim Bending force

Excited mode depends on $|\dot{\gamma}|$

Particle larger than the integral length

Correlation depends on the large scale flow

Particle larger than the integral length

Correlation depends on the large scale flow

In homogenous isotropic turbulence

$$\langle u(x + \ell, t + \tau)u(x, t) \rangle \sim \delta(\ell)\delta(\tau) \quad \text{for} \quad \ell \gg L_I \quad \text{and} \quad \tau \gg T_I$$

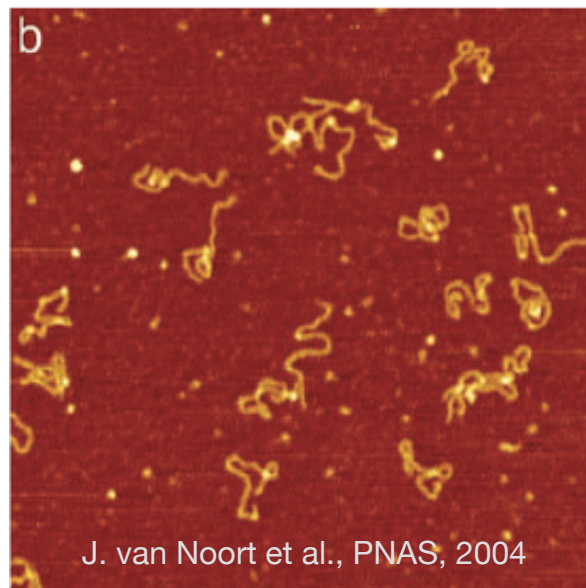
Particle larger than the integral length

Correlation depends on the large scale flow

In homogenous isotropic turbulence

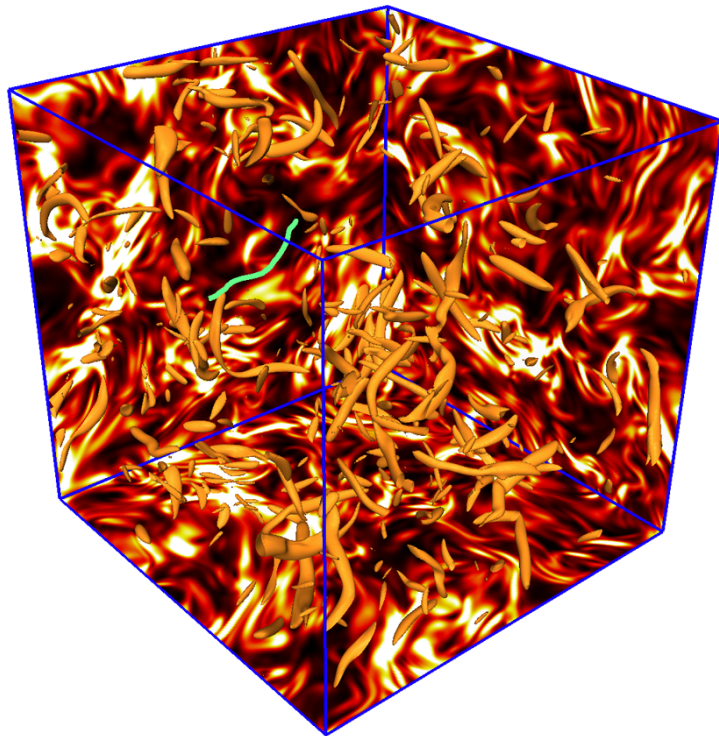
$$\langle u(x + \ell, t + \tau)u(x, t) \rangle \sim \delta(\ell)\delta(\tau) \quad \text{for} \quad \ell \gg L_I \quad \text{and} \quad \tau \gg T_I$$

Deformations are expected to be similar to polymer

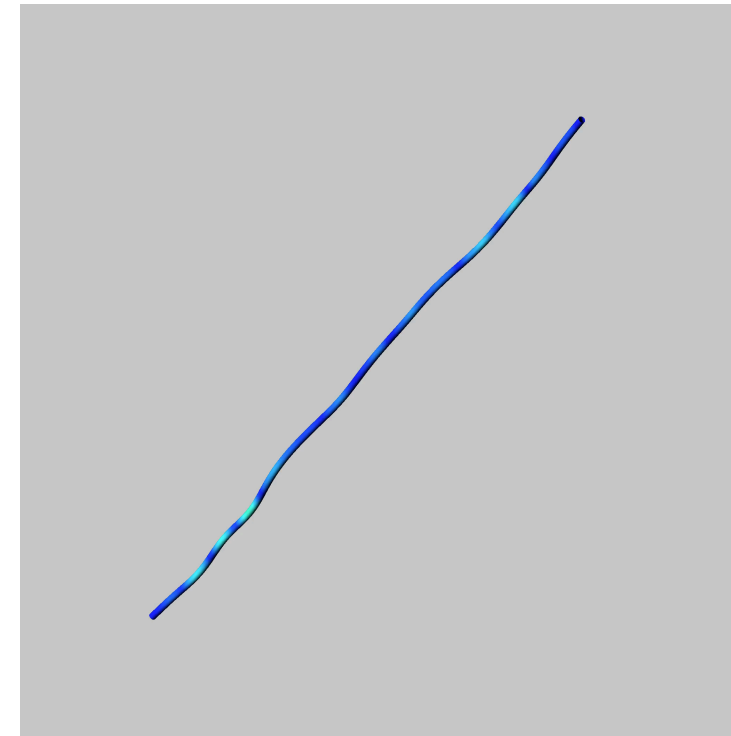


Particle in the inertial range

- Preferential alignment?
- Several length scales and timescales



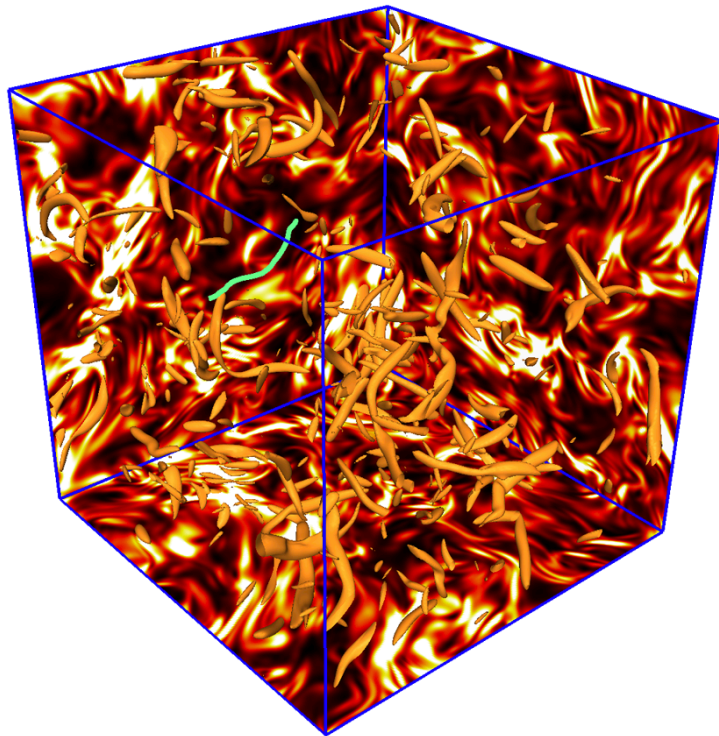
Rosti *et al.*, Phys. Rev. Lett., **121**(4), 044501, 2018



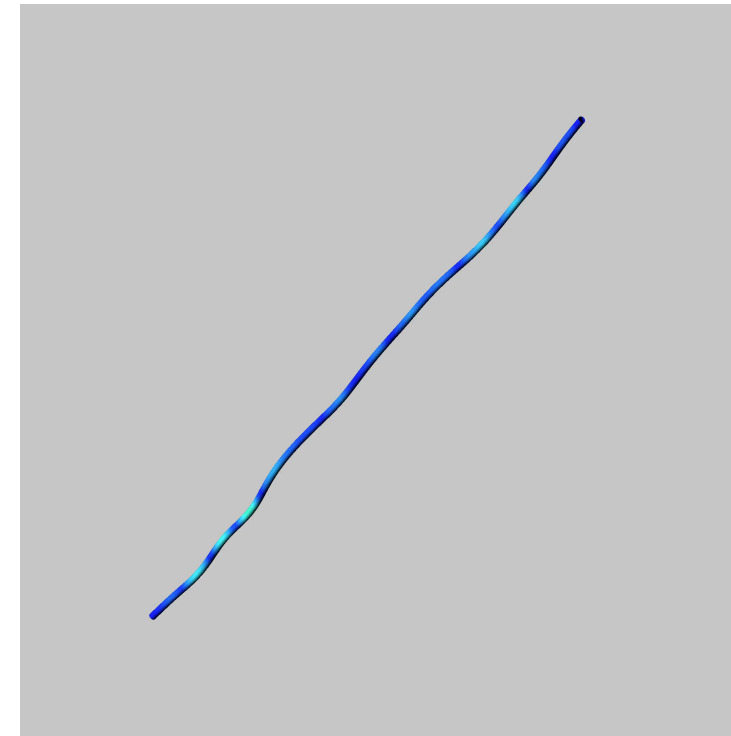
KS (B. Favier)

Particle in the inertial range

- Preferential alignment?
- Several length scales and timescales

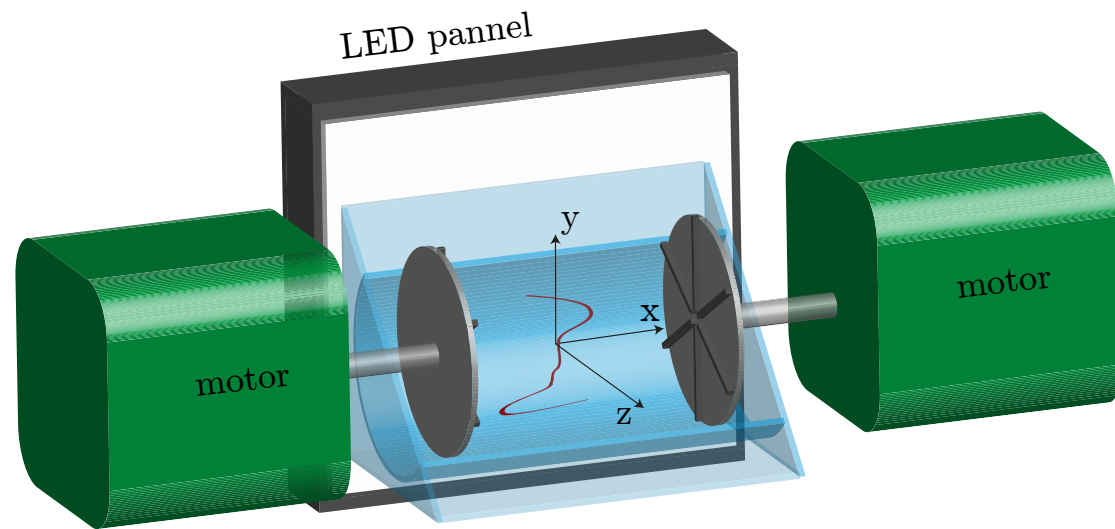


Rosti *et al.*, Phys. Rev. Lett., **121**(4), 044501, 2018



KS (B. Favier)

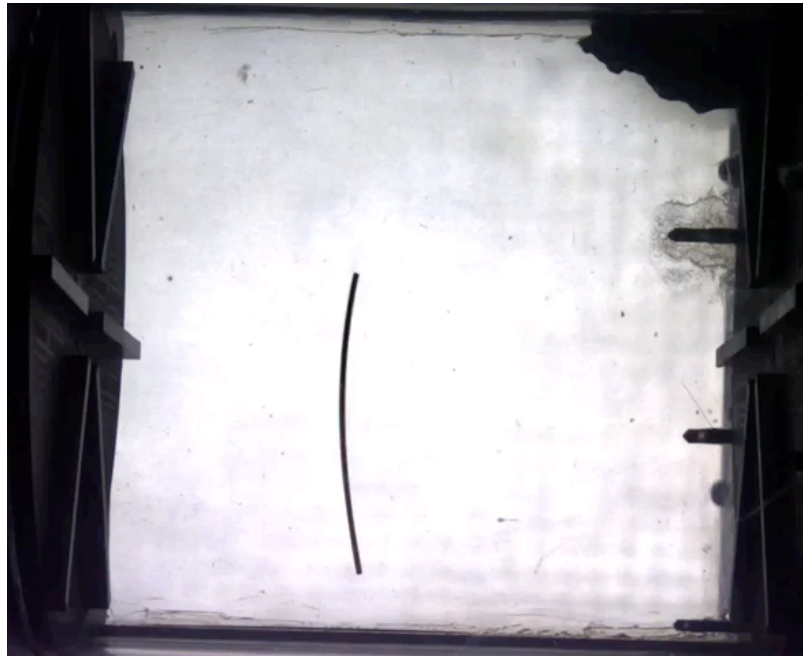
When does it bend?



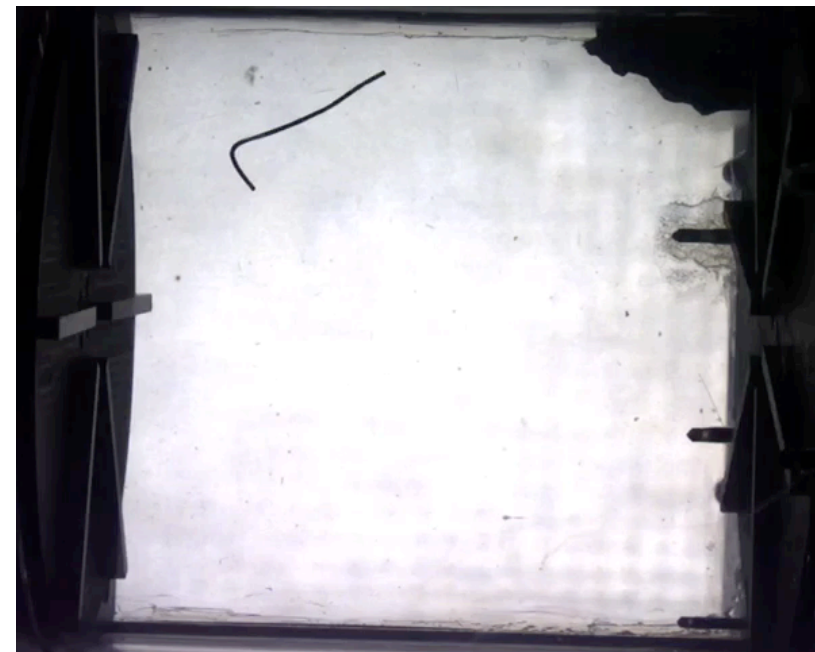
$$R \sim 8.2 \text{ cm}$$
$$Re \in [10^5 ; 10^6]$$
$$\eta_K \in [12 ; 91] \mu\text{m}$$



Video camera

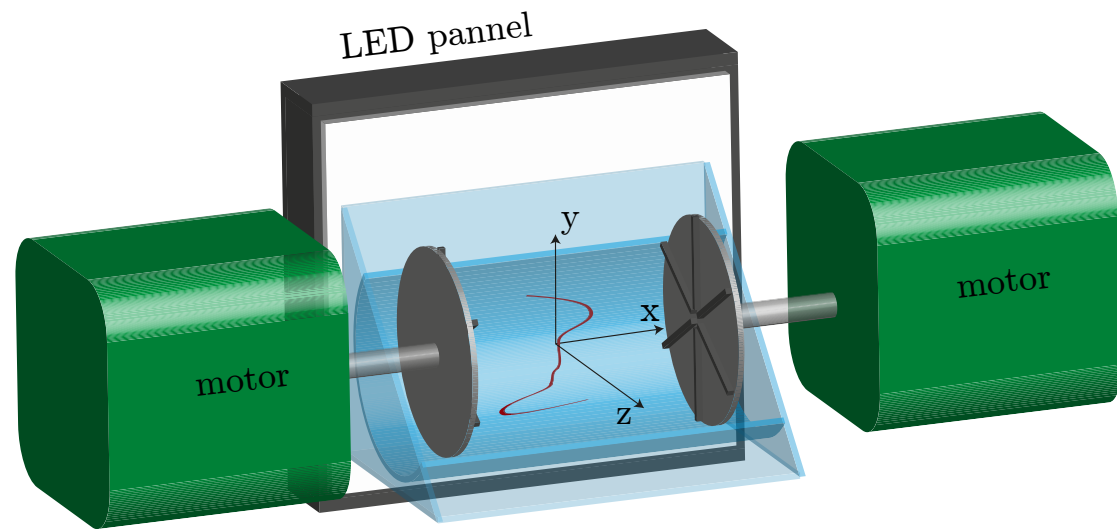


F=2 Hz



F=20 Hz

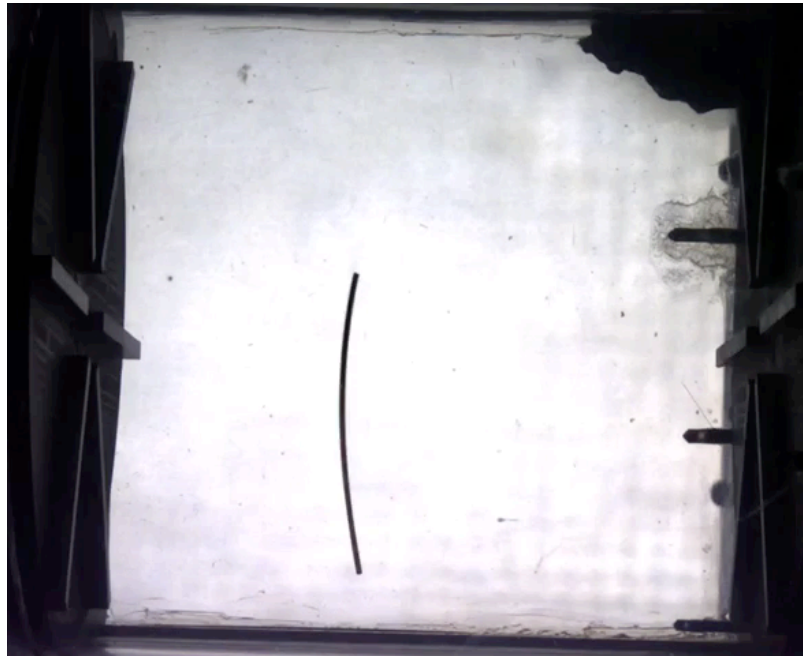
When does it bend?



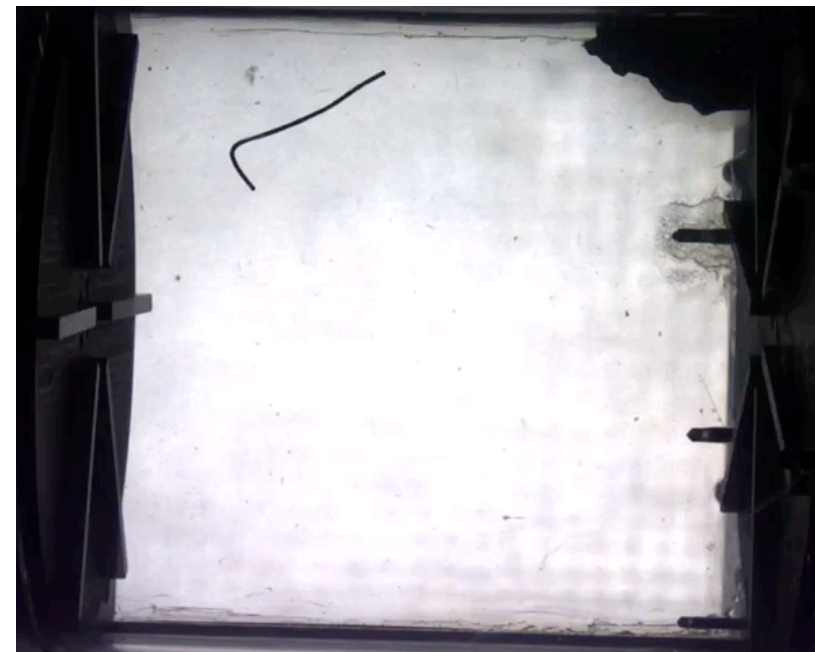
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$$\eta_K \in [12 ; 91] \mu\text{m}$$



Video camera

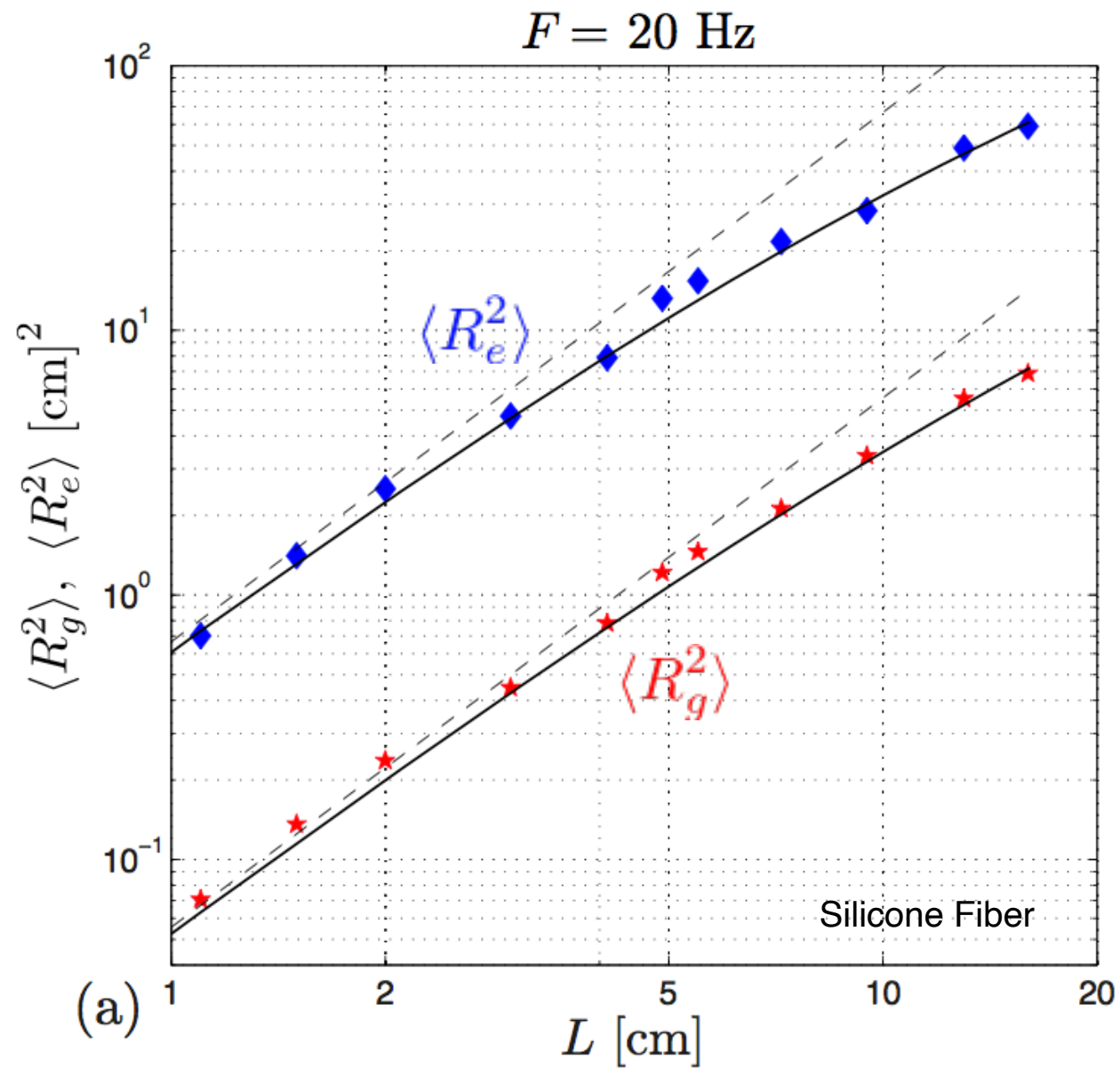


F=2 Hz

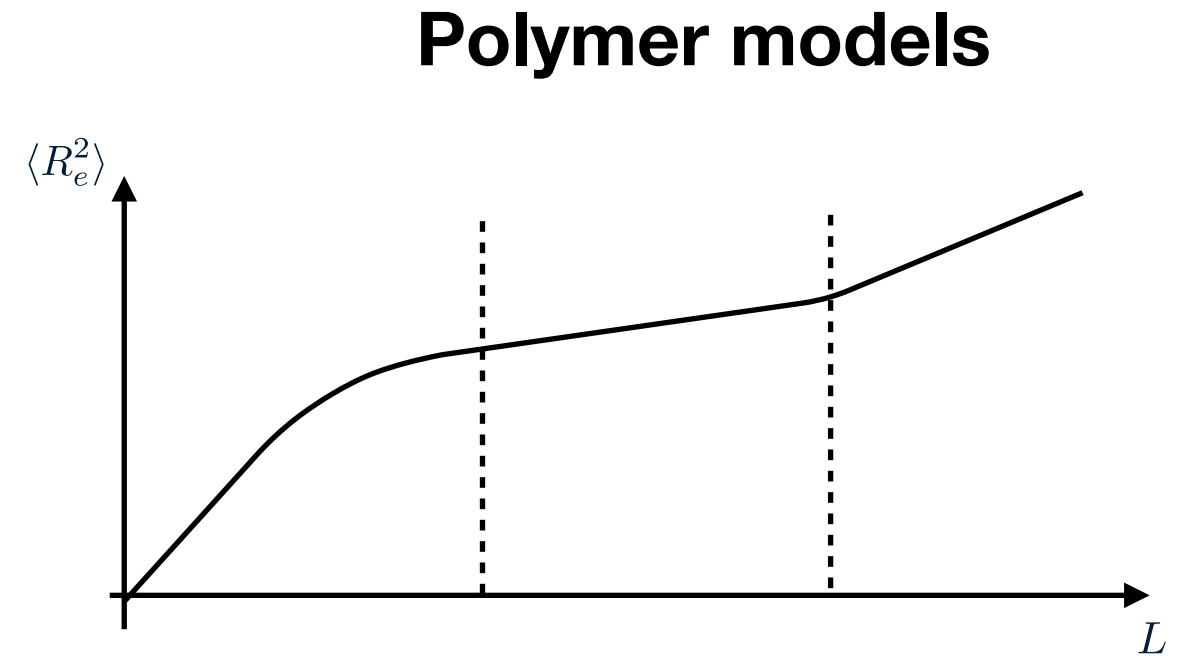
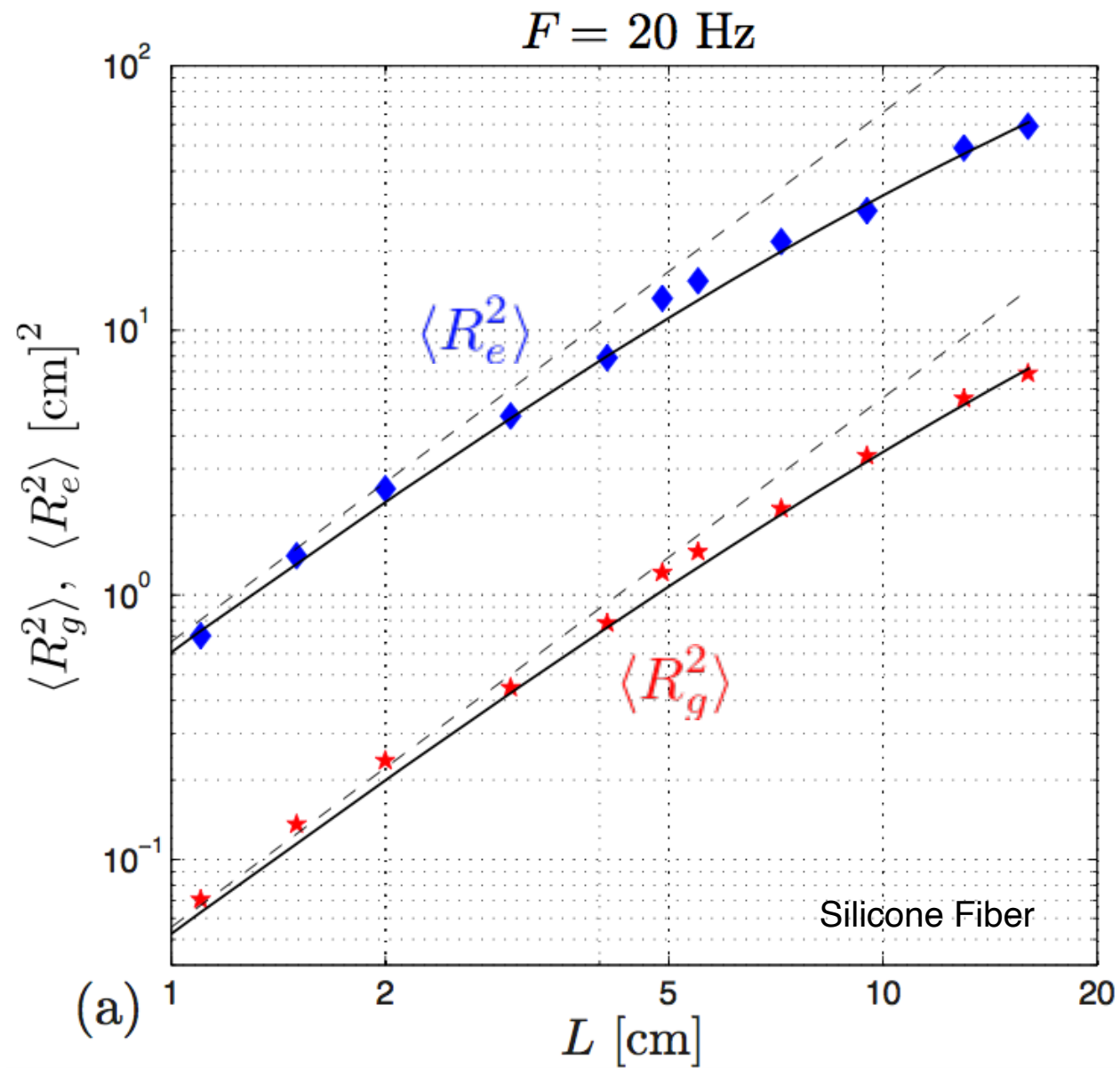


F=20 Hz

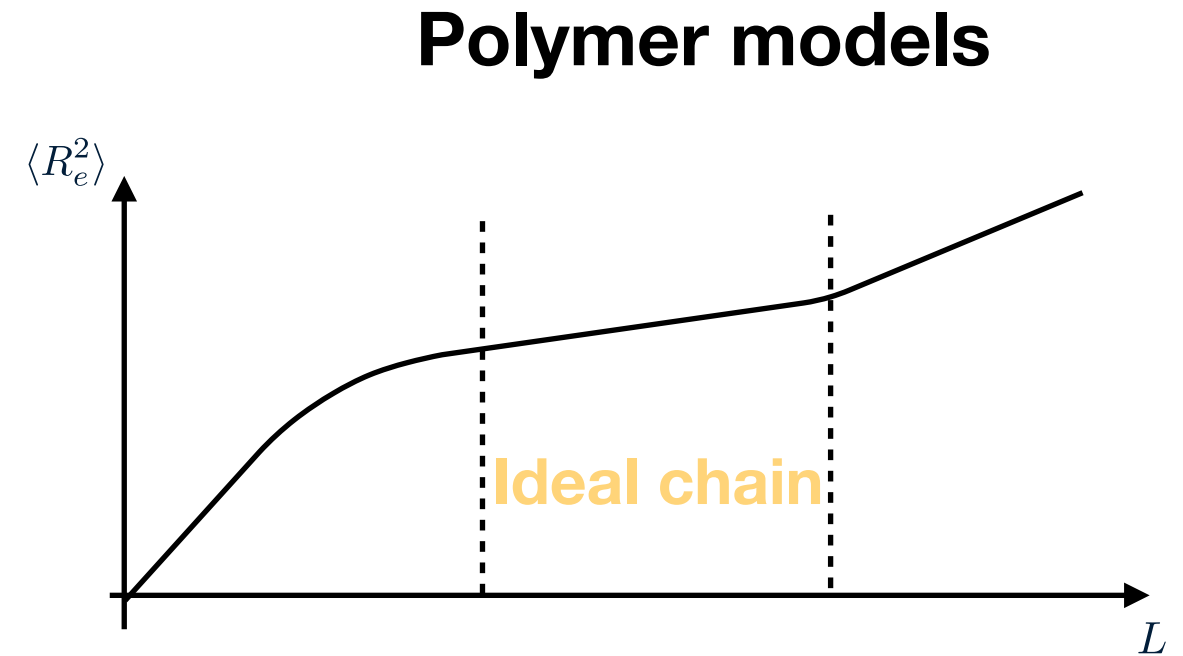
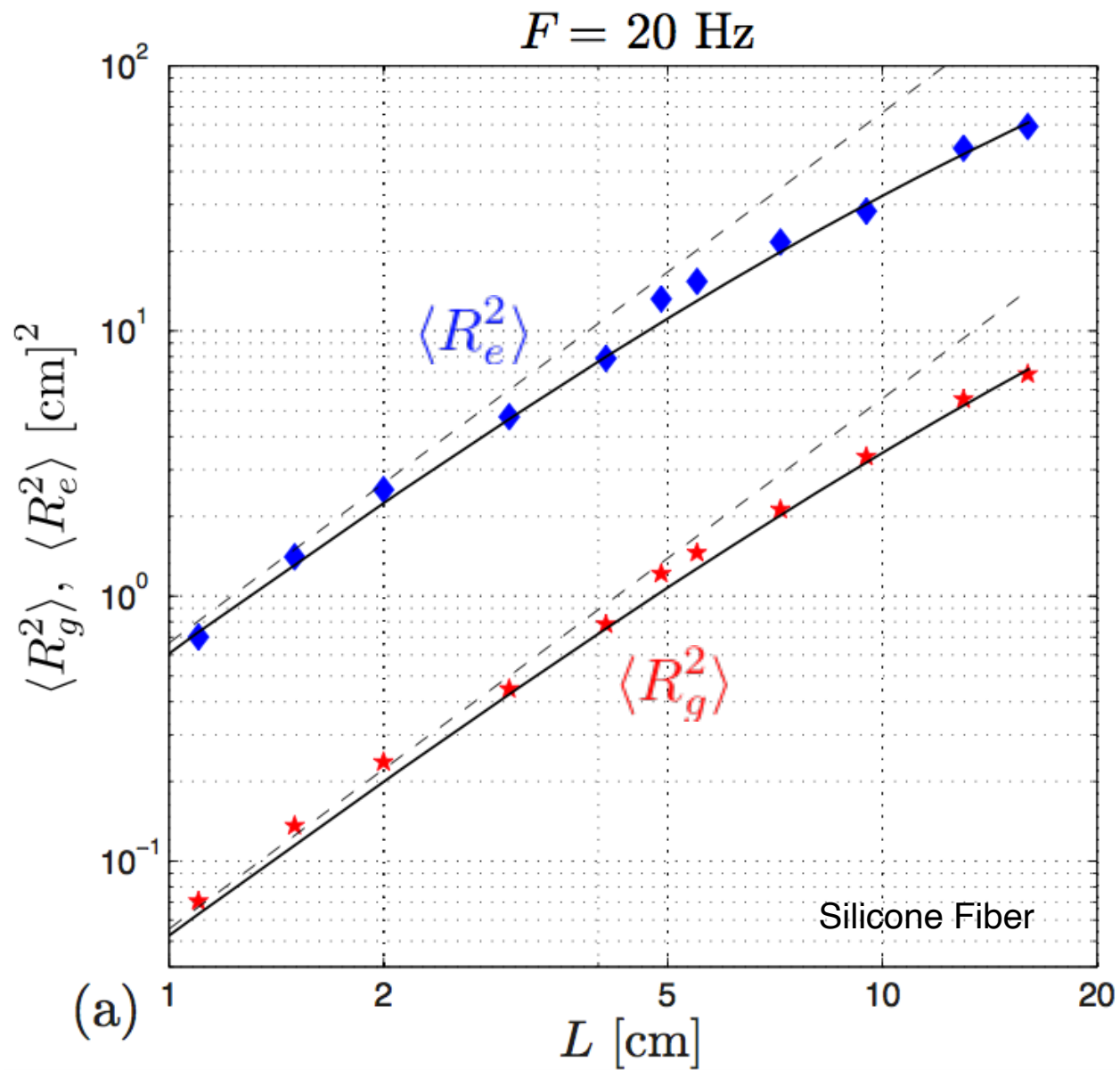
Evolution of the end-to-end vector



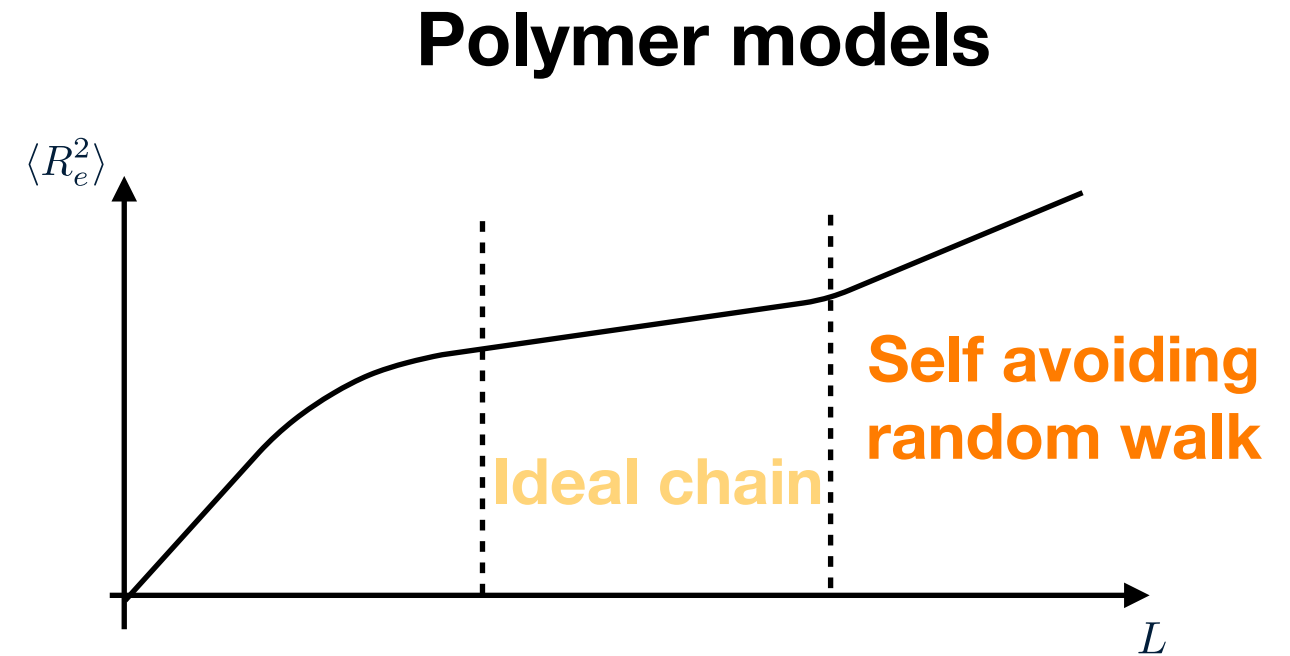
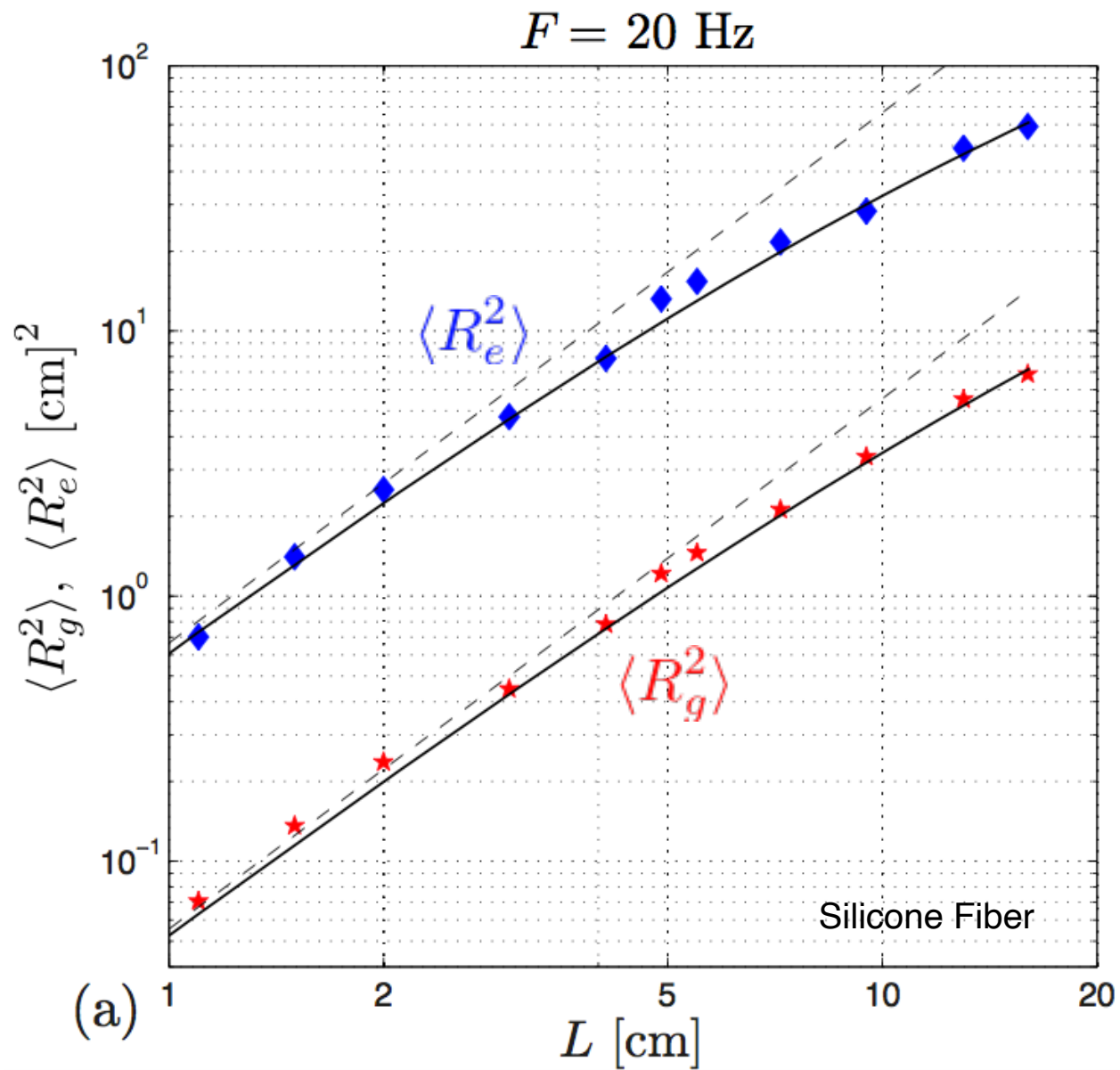
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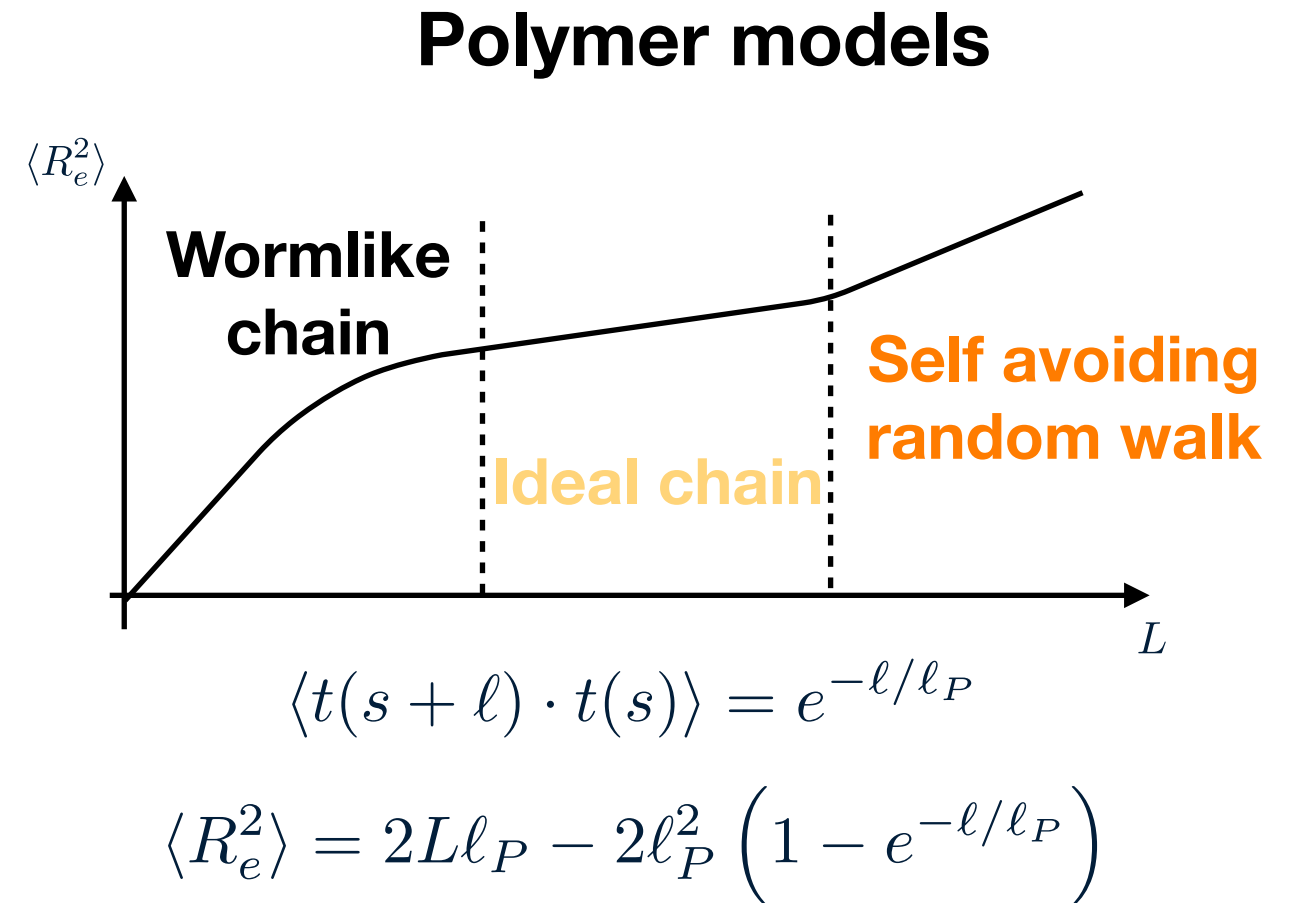
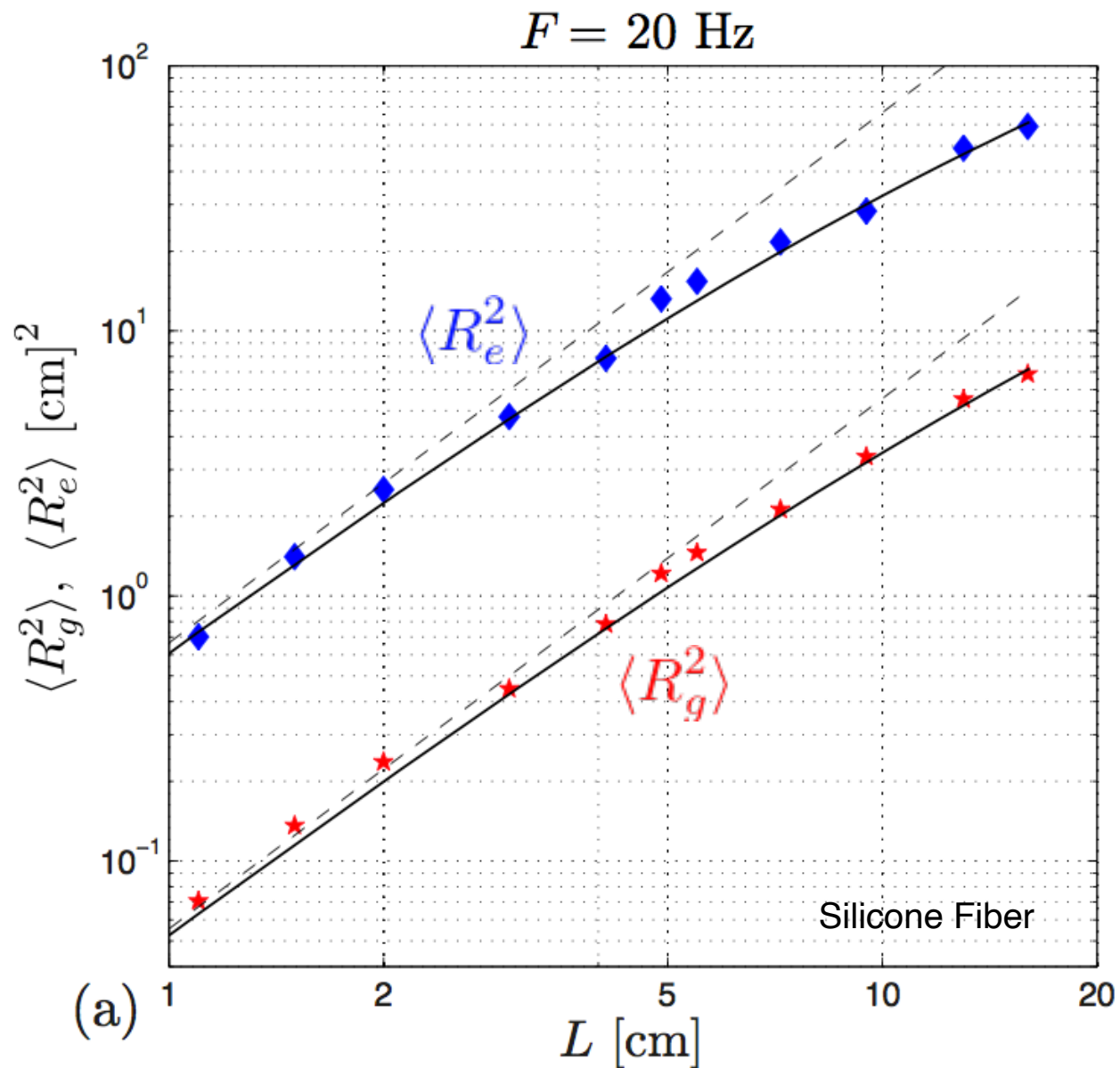
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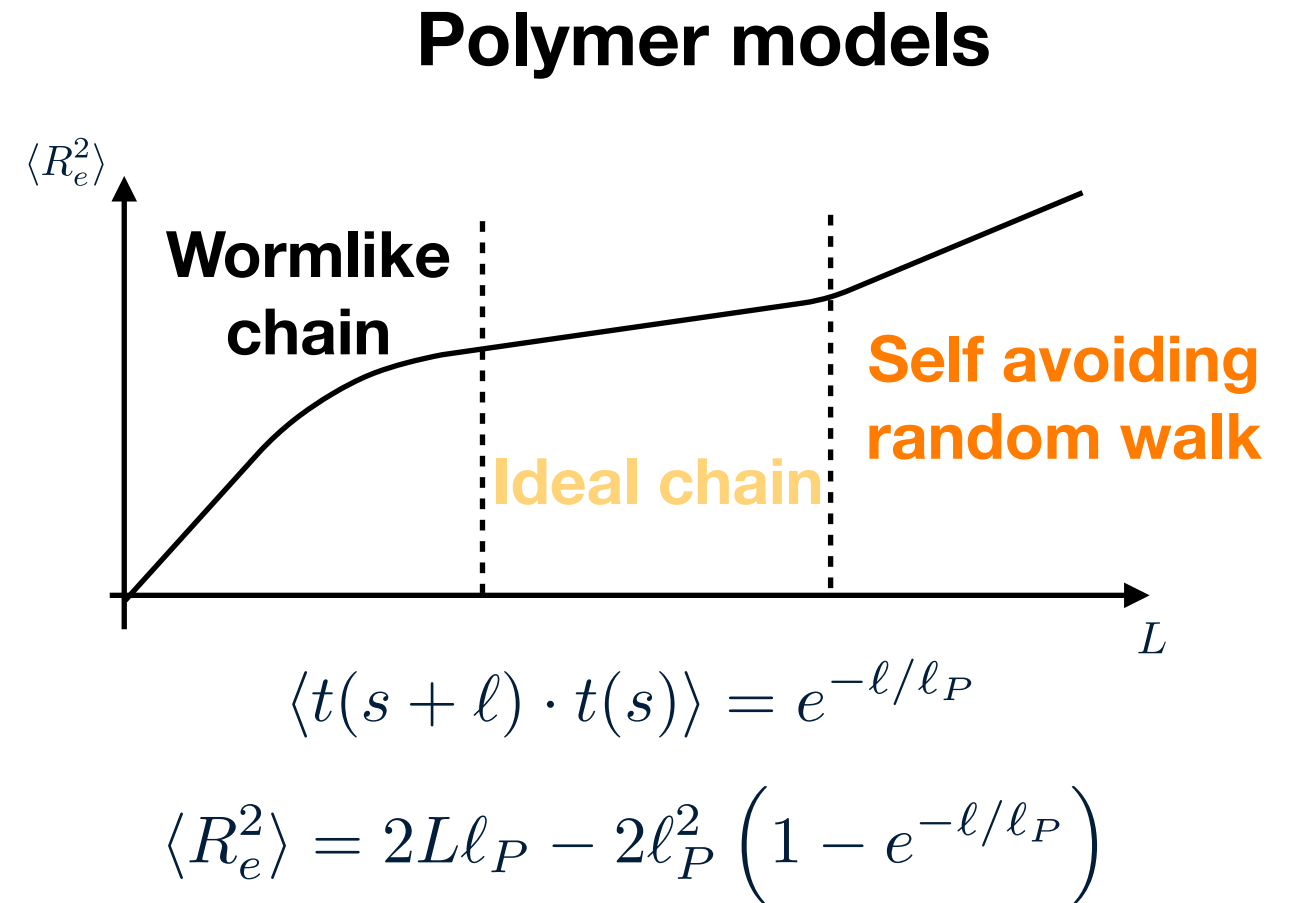
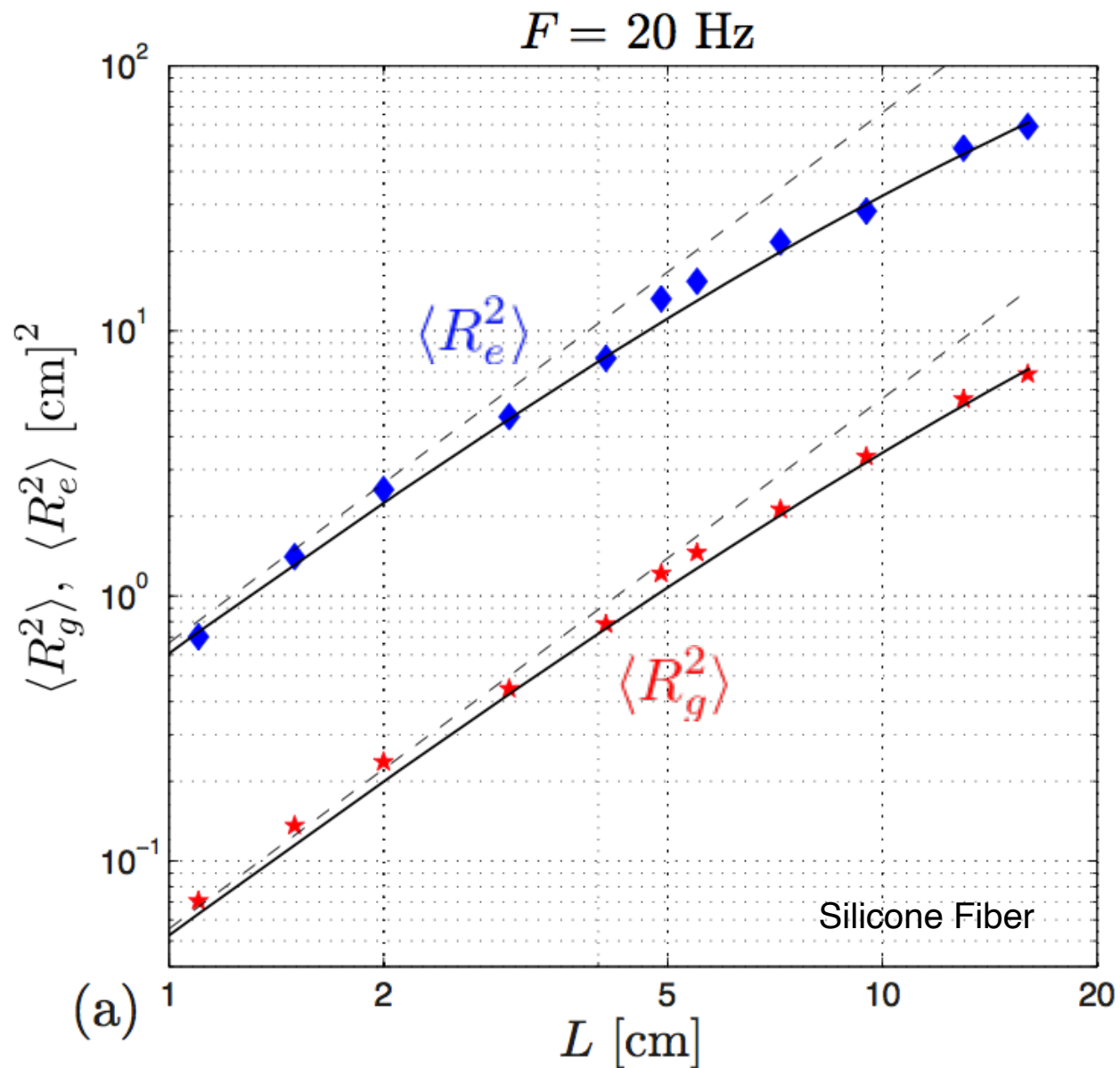
Evolution of the end-to-end vector



Evolution of the end-to-end vector

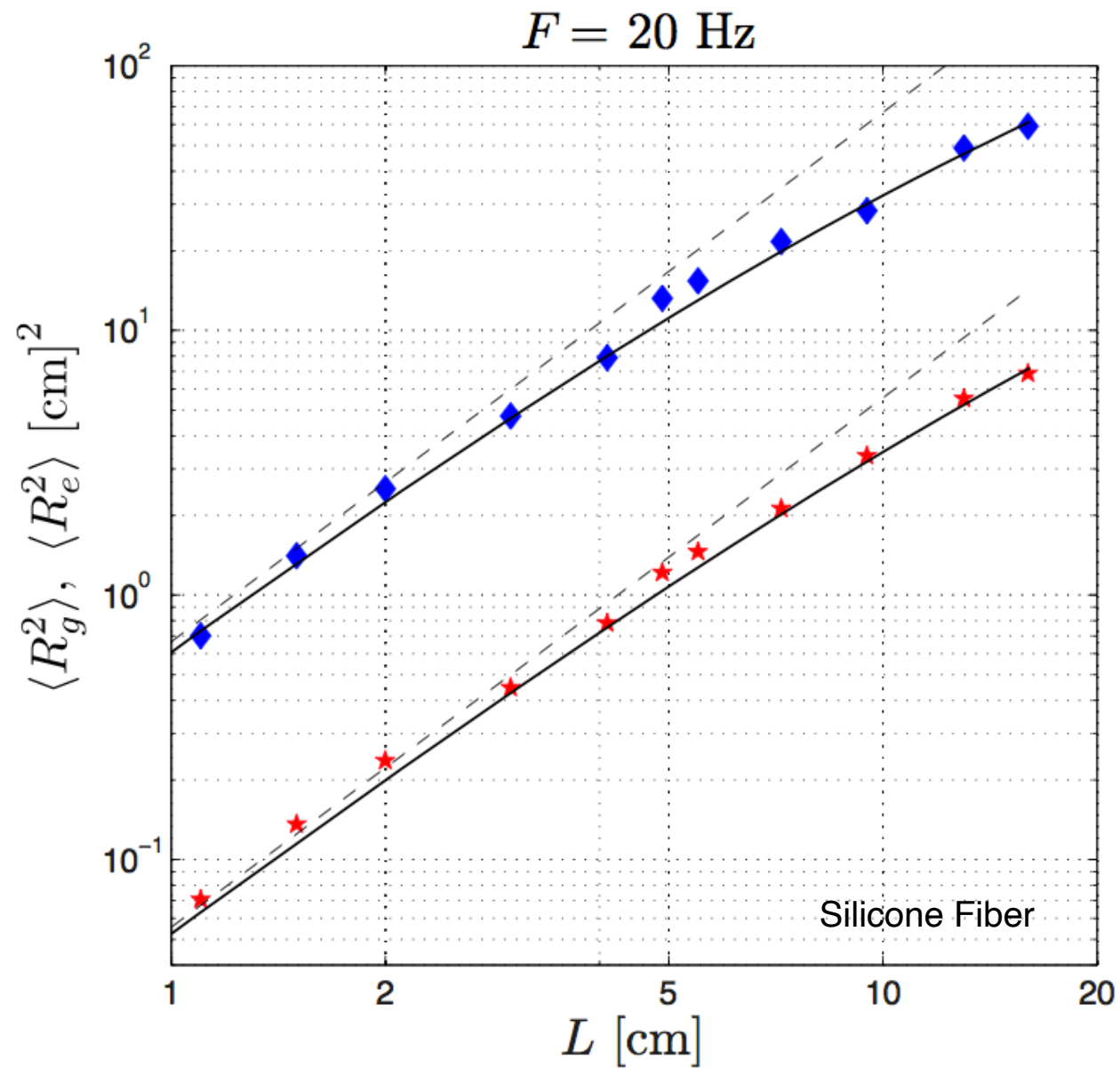


Evolution of the end-to-end vector

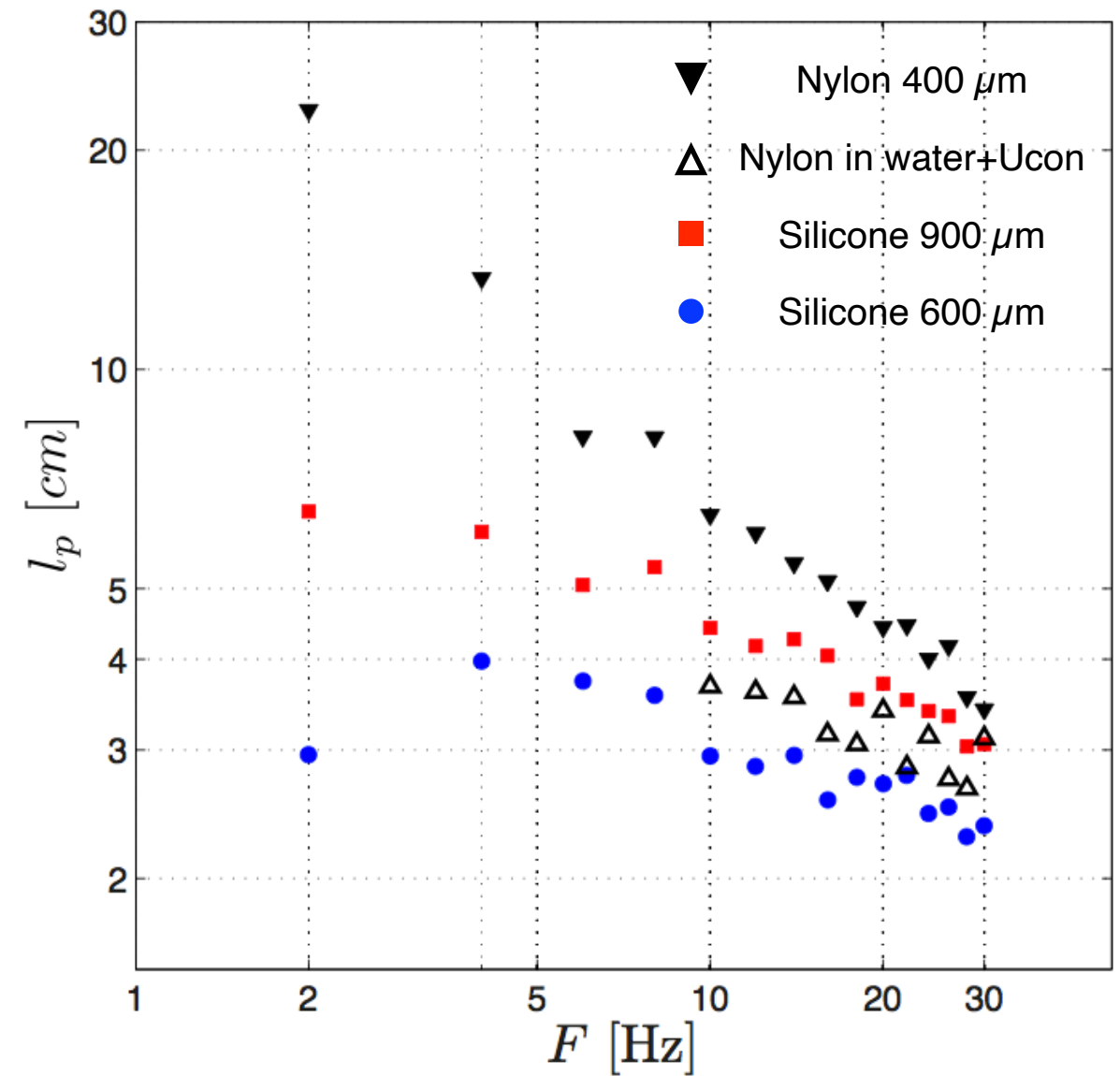
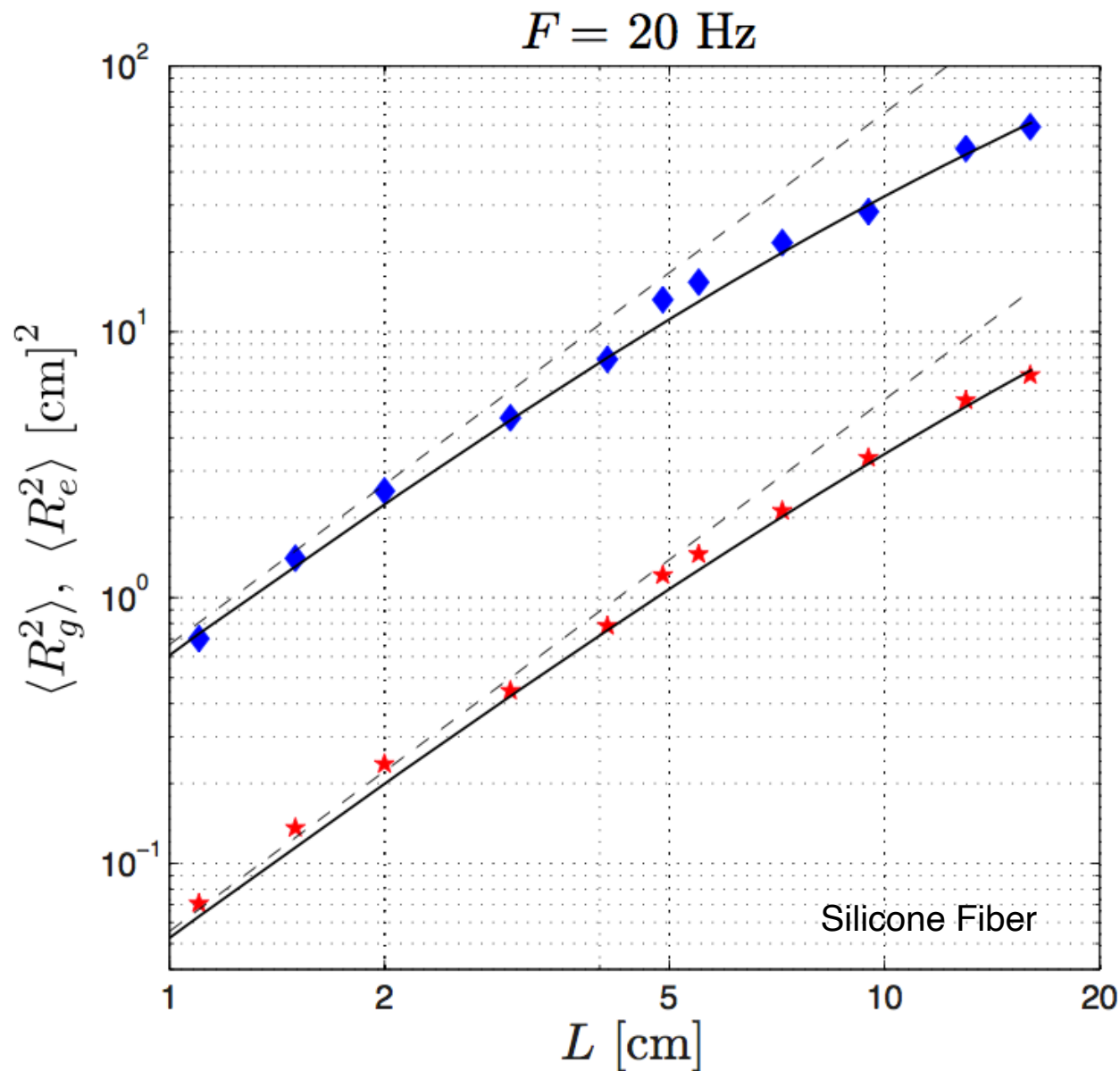


The physics is hidden in ℓ_P

Evolution of the persistence length



Evolution of the persistence length



Fluid	dyn. visc. [mPa.s]	density	Re
water	1	1	$10^5 - 1.5 \cdot 10^6$
water+Ucon	100 ± 20	1,045	$5 \cdot 10^3 - 1.5 \cdot 10^4$

Transition from rigid to flexible particles

Fibers

In polymer theory: $\frac{EI}{\ell_p} = k_B T$

Transition from rigid to flexible particles

Fibers

In polymer theory: $\frac{EI}{\ell_p} = k_B T$

Analogy in turbulence

$$\ell \sim \left(\frac{EI}{\rho} \right)^{3/14} \varepsilon^{-1/7}$$

Independent of the viscosity

Transition from rigid to flexible particles

Fibers

In polymer theory: $\frac{EI}{\ell_p} = k_B T$

Analogy in turbulence

$$\ell \sim \left(\frac{EI}{\rho} \right)^{3/14} \varepsilon^{-1/7}$$

Independent of the viscosity

Out of equilibrium stationary process

$$\partial_t \mathcal{E} = P_{inj} - P_{dis}$$

$$P_{inj} \sim \rho \ell^3 \varepsilon \quad P_{dis} \sim \frac{EI}{\ell \tau_{el}}$$

Transition from rigid to flexible particles

Fibers

In polymer theory: $\frac{EI}{\ell_p} = k_B T$

Analogy in turbulence

$$\ell \sim \left(\frac{EI}{\rho} \right)^{3/14} \varepsilon^{-1/7}$$

Independent of the viscosity

Out of equilibrium stationary process

$$\partial_t \mathcal{E} = P_{inj} - P_{dis}$$

$$P_{inj} \sim \rho \ell^3 \varepsilon \quad P_{dis} \sim \frac{EI}{\ell \tau_{el}}$$

Deformation timescale

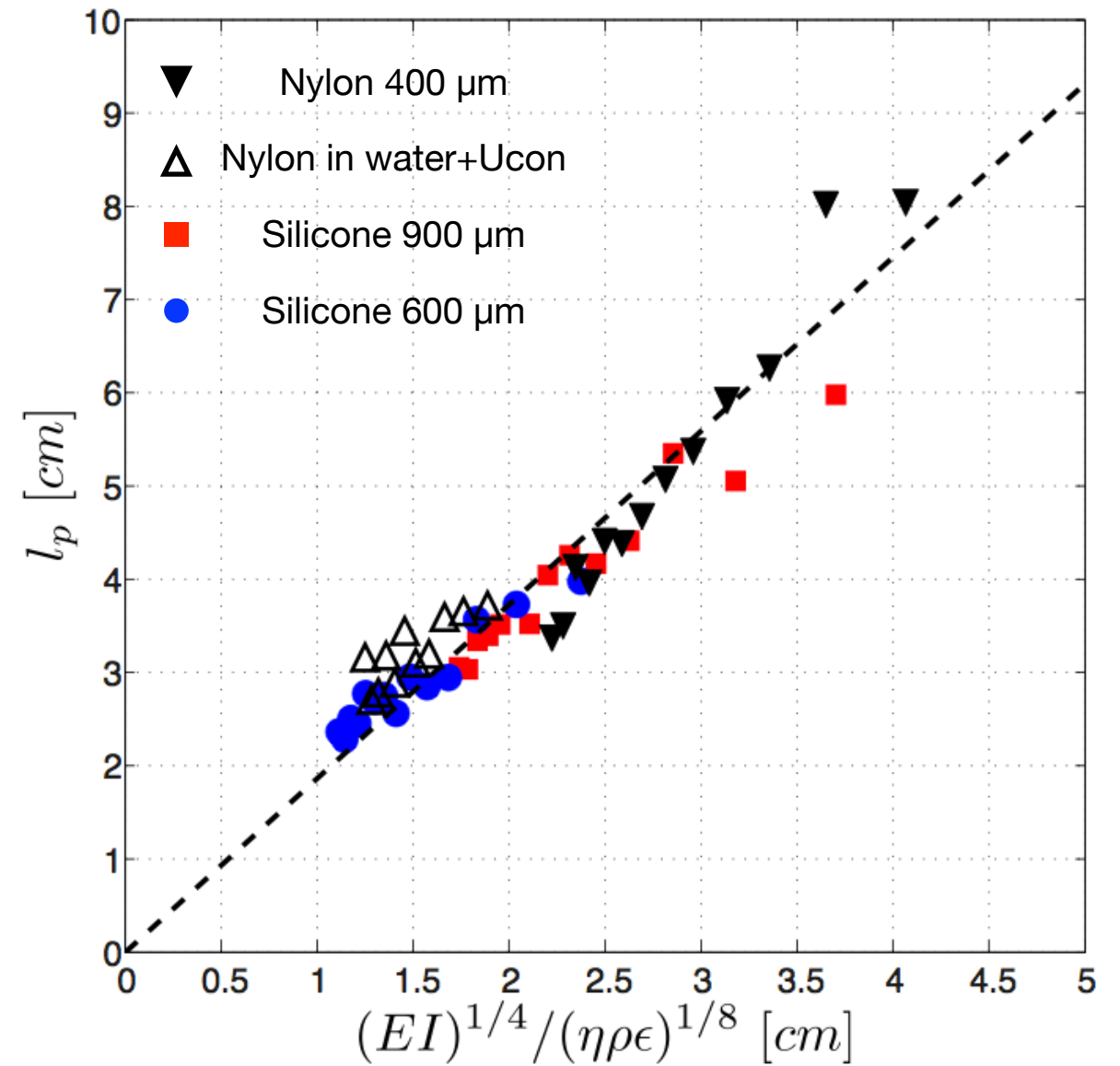
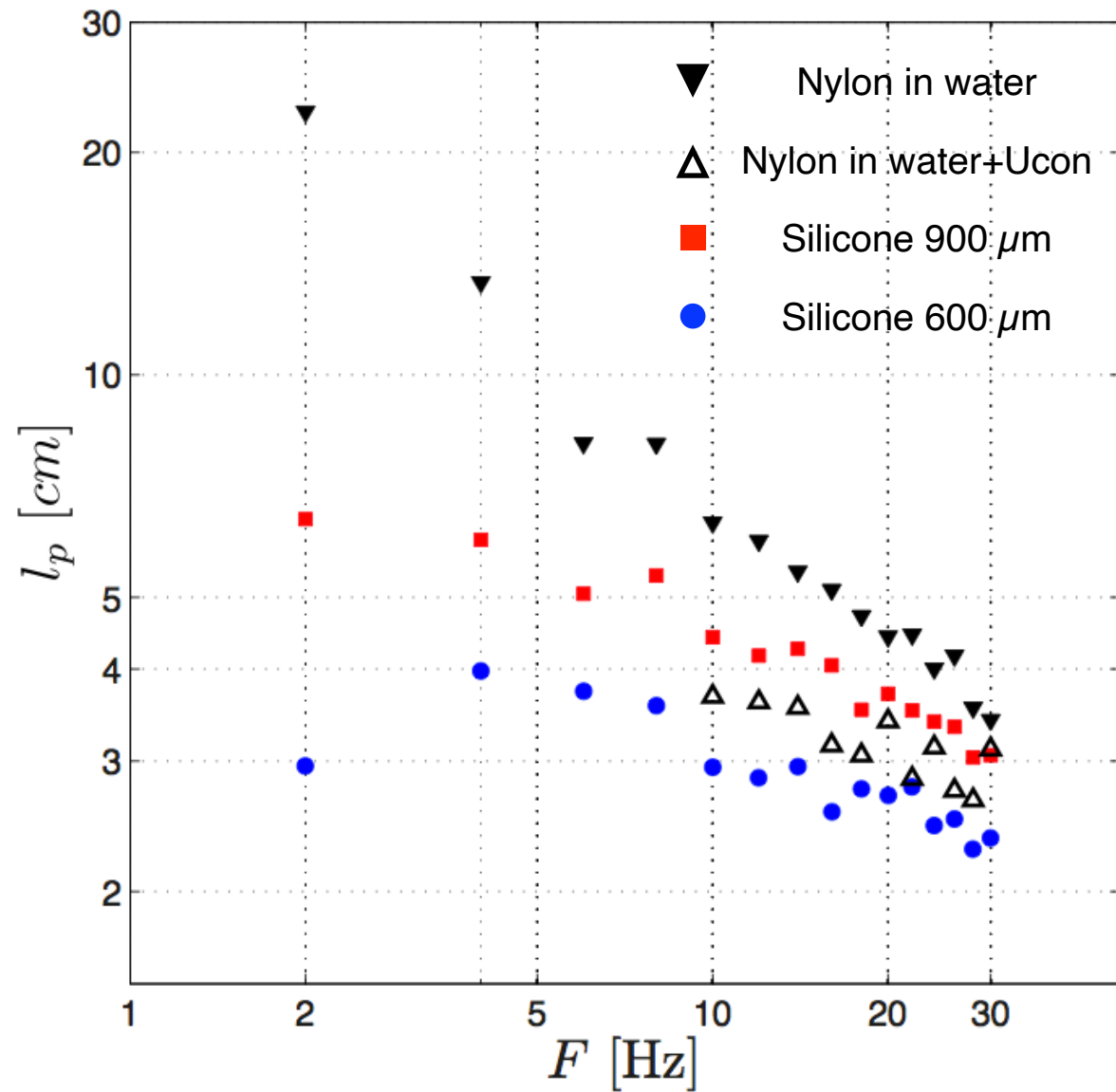
$$EI \partial_s^4 \mathbf{r} \sim \eta \partial_t \mathbf{r}$$

$$\tau_{el} \sim \frac{\eta L^4}{EI}$$

Balance of power

$$\ell_e \sim \frac{(EI)^{1/4}}{(\rho_f \eta \varepsilon)^{1/8}}$$

Modeling the persistence length



$$l_e \sim \frac{(EI)^{1/4}}{(\rho\eta\epsilon)^{1/8}}$$

Characterization of fiber deformations

Characterization of fiber deformations

Numerical simulation

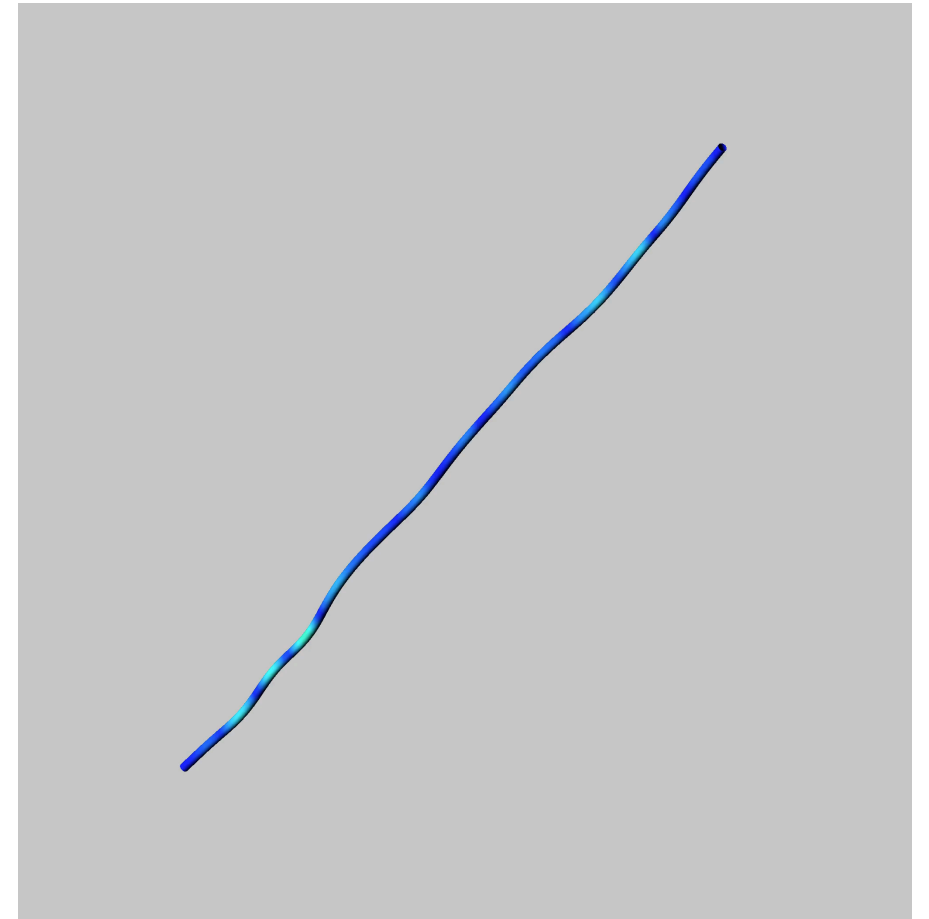
Elasticity

$$\partial_{tt}\mathbf{r} - \partial_s(T\partial_s\mathbf{r}) + \frac{1}{St}\partial_t\mathbf{r} + \gamma\partial_s^4\mathbf{r} = \frac{1}{St}\mathbf{u}$$

$$|\partial_s\mathbf{r}|^2 = 1 \quad \gamma = \frac{EI}{\sigma u^2 L_I^2} \quad St = \frac{\sigma U}{4\pi\eta L_I}$$

Flow: kinematic simulation

Curvature $\kappa^2 = |\partial_{ss}\mathbf{r}|^2$



Characterization of fiber deformations

Numerical simulation

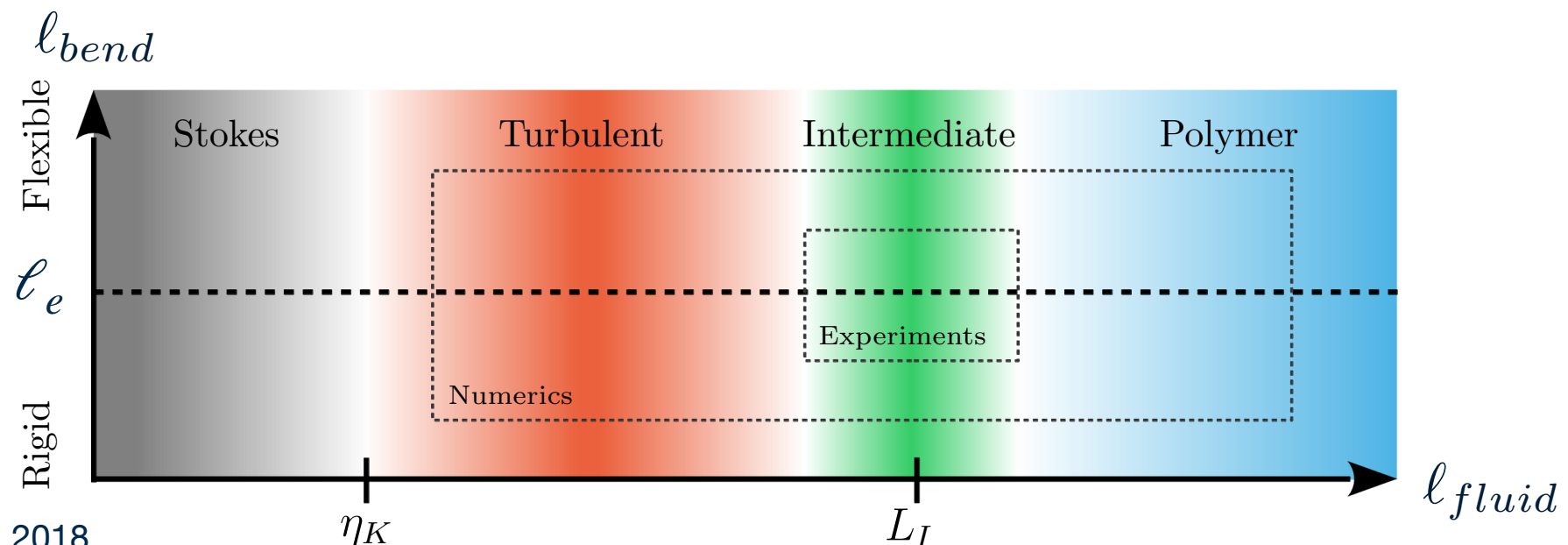
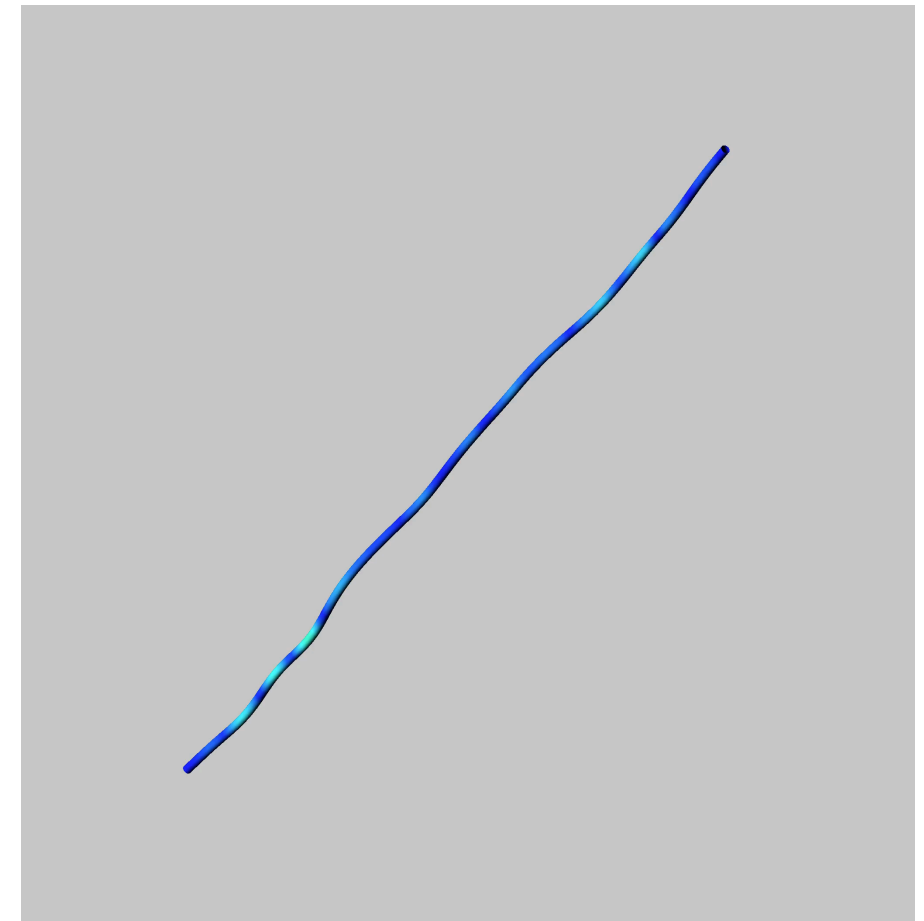
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Characterization of fiber deformations

Curvature $\kappa^2 = |\partial_{ss}\mathbf{r}|^2$

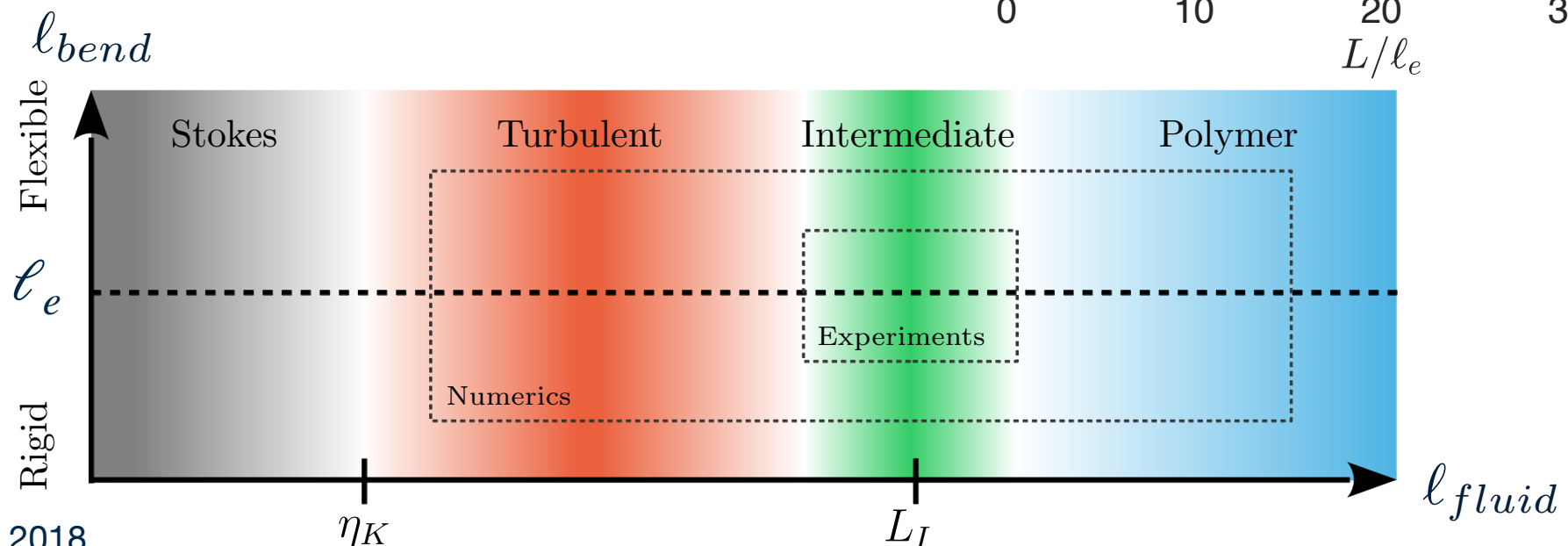
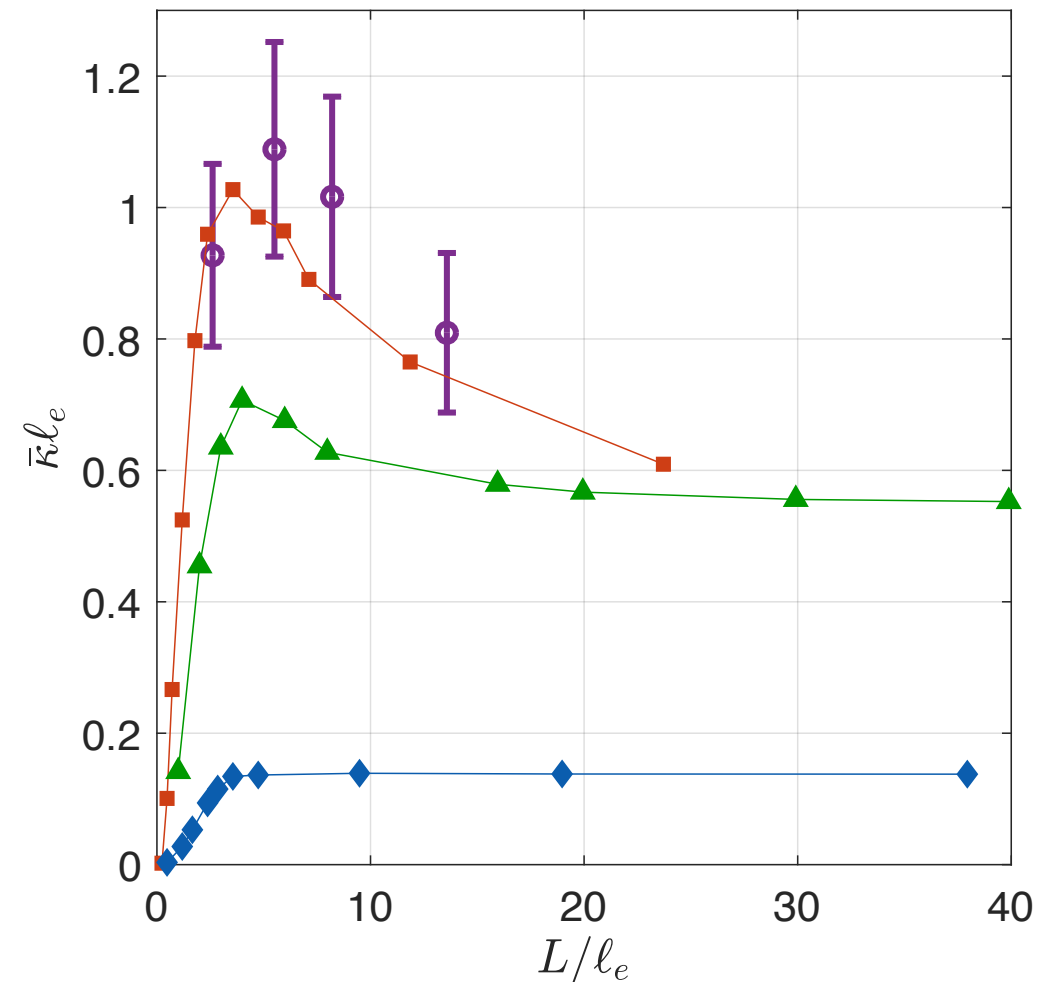
Numerical simulation

Elasticity

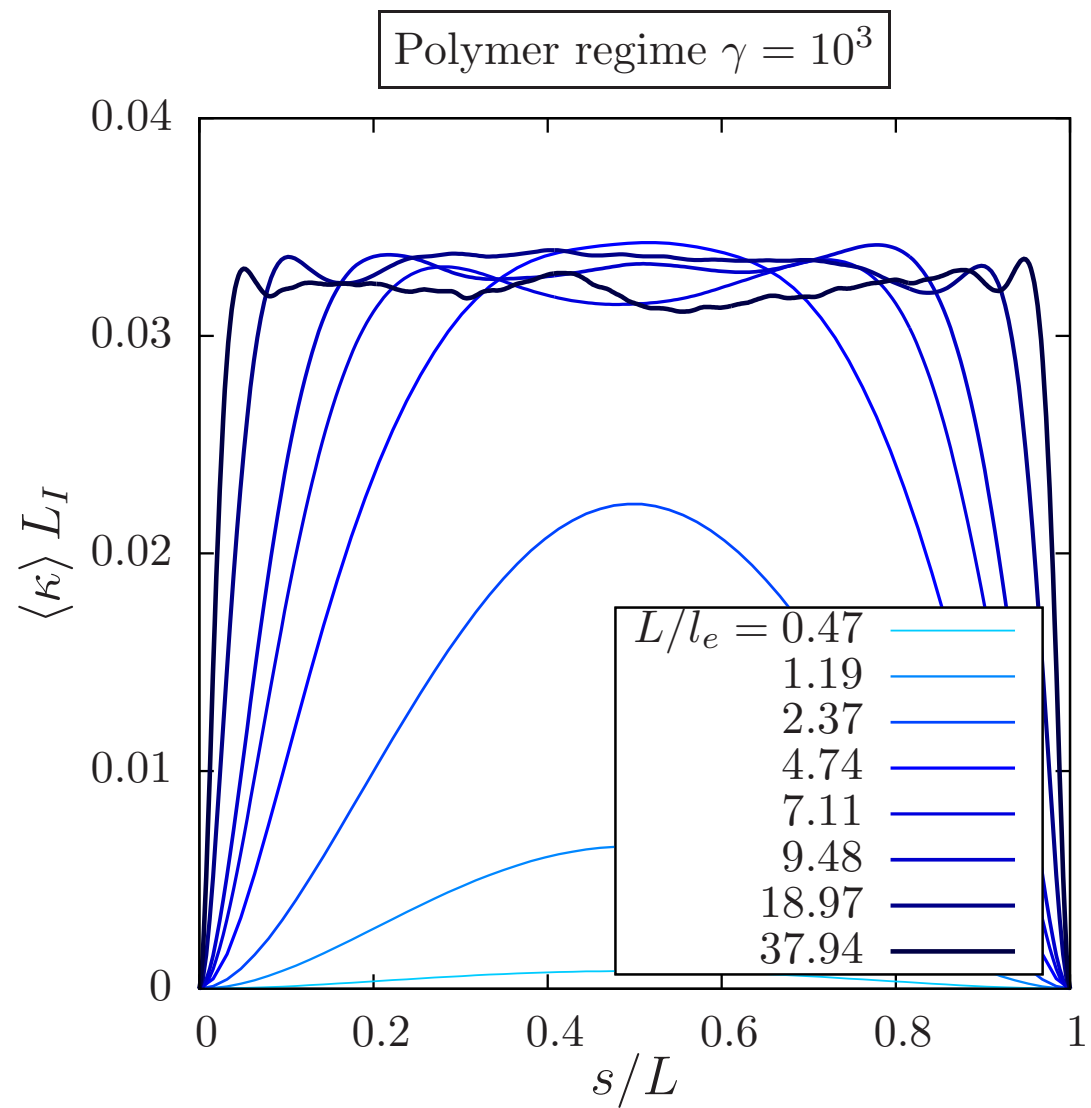
$$\partial_{tt}\mathbf{r} - \partial_s(T\partial_s\mathbf{r}) + \frac{1}{St}\partial_t\mathbf{r} + \gamma\partial_s^4\mathbf{r} = \frac{1}{St}\mathbf{u}$$

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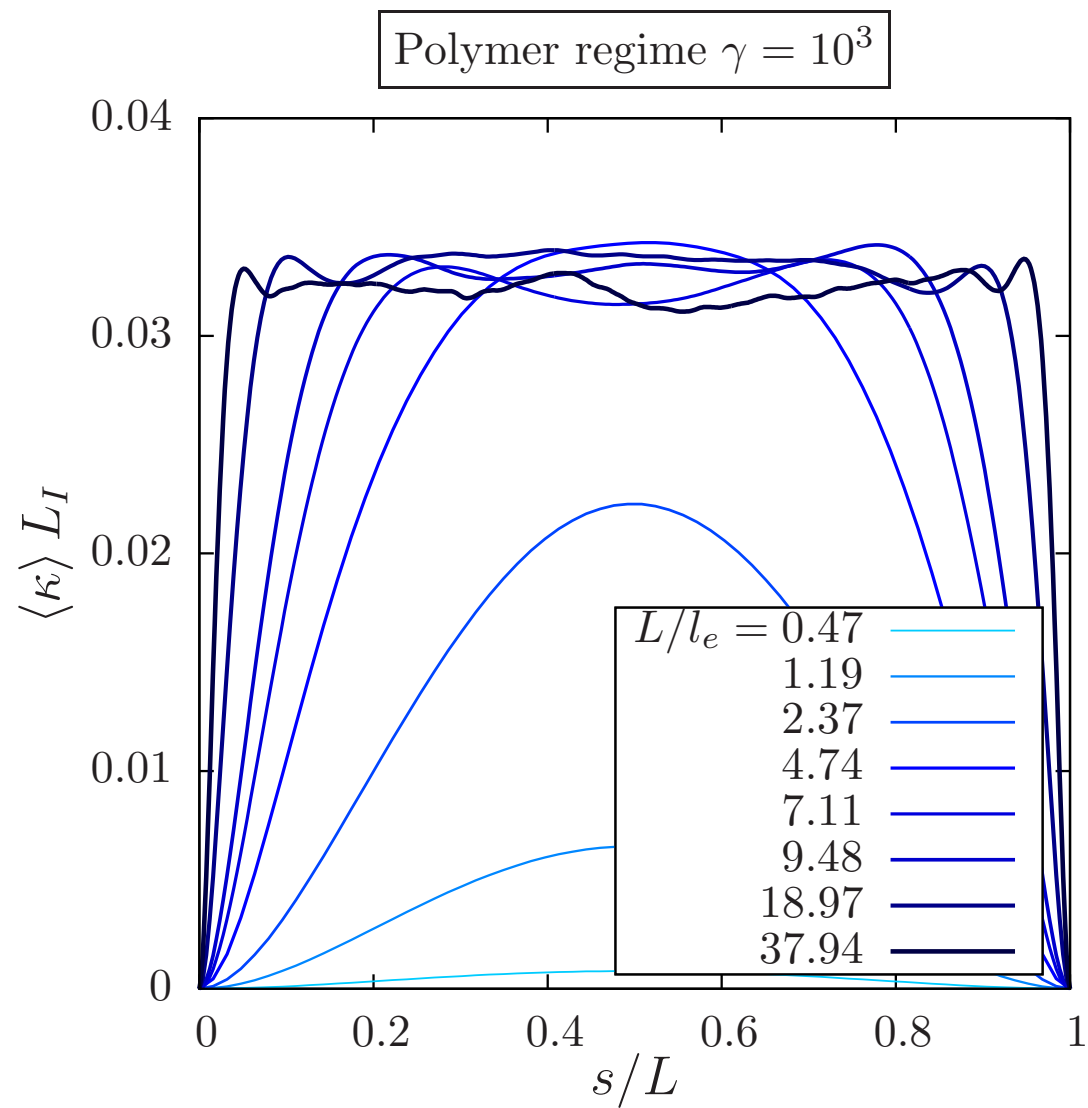


Spatial evolution of the local curvature

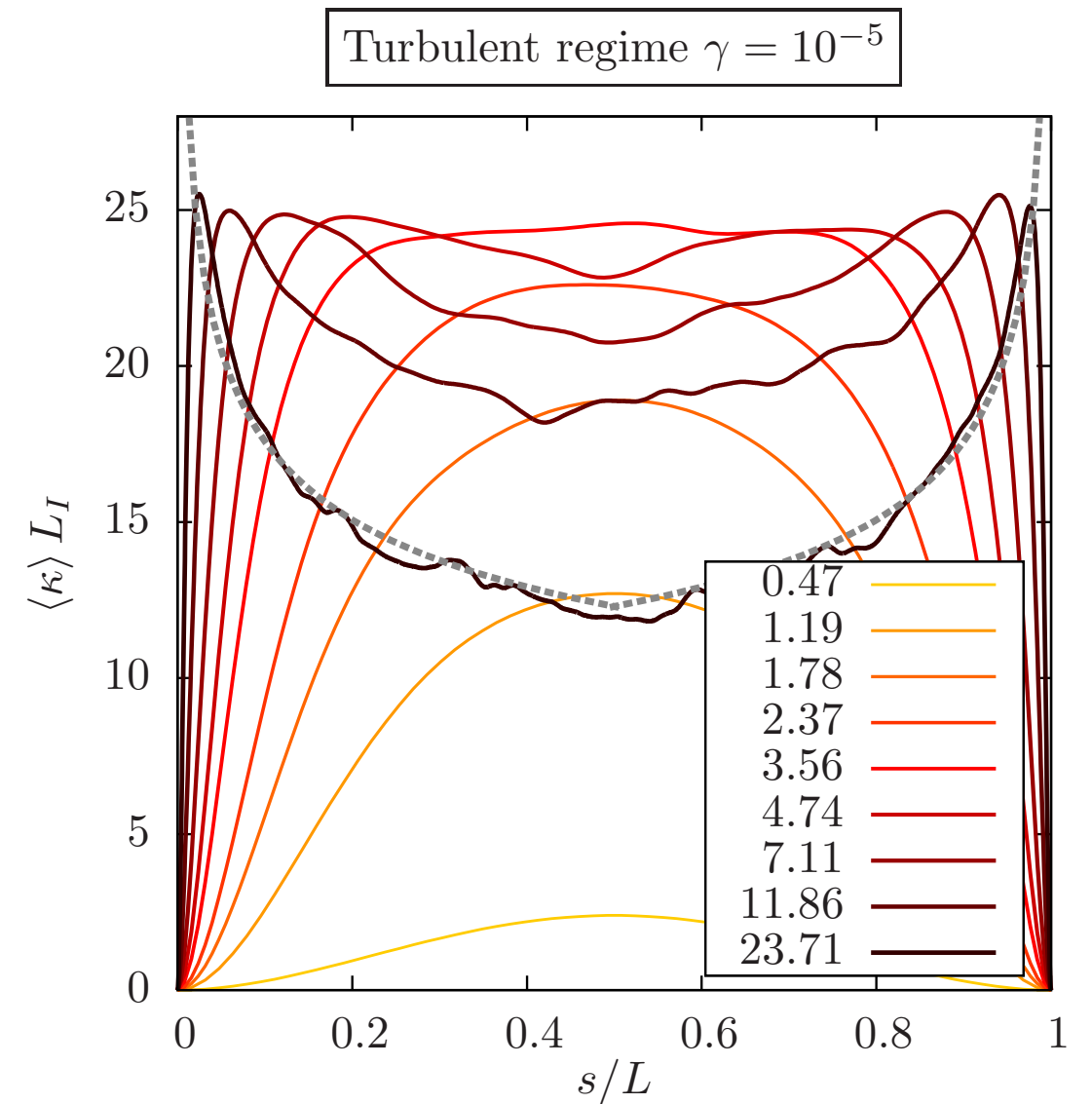


Polymer regime

Spatial evolution of the local curvature

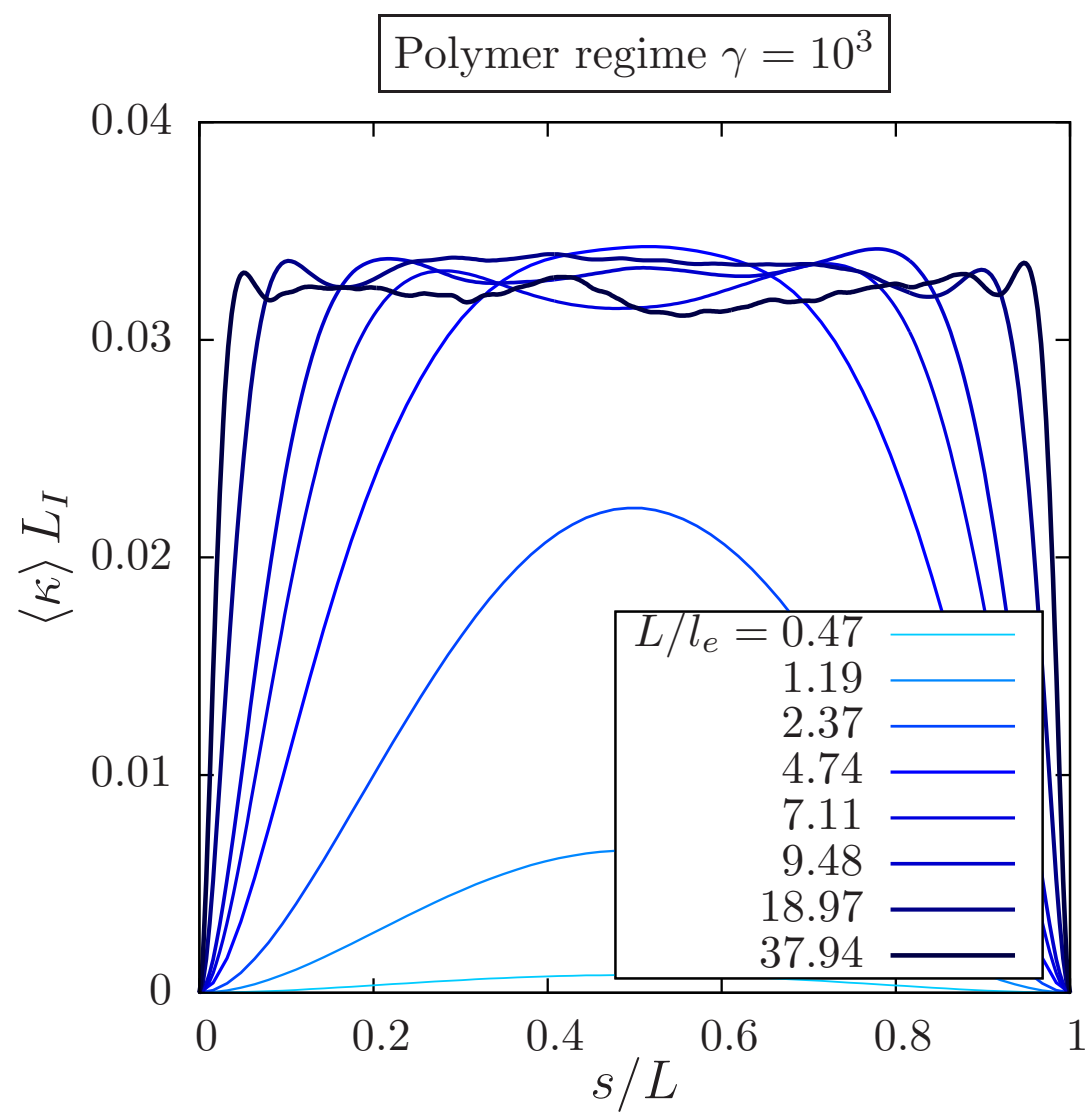


Polymer regime

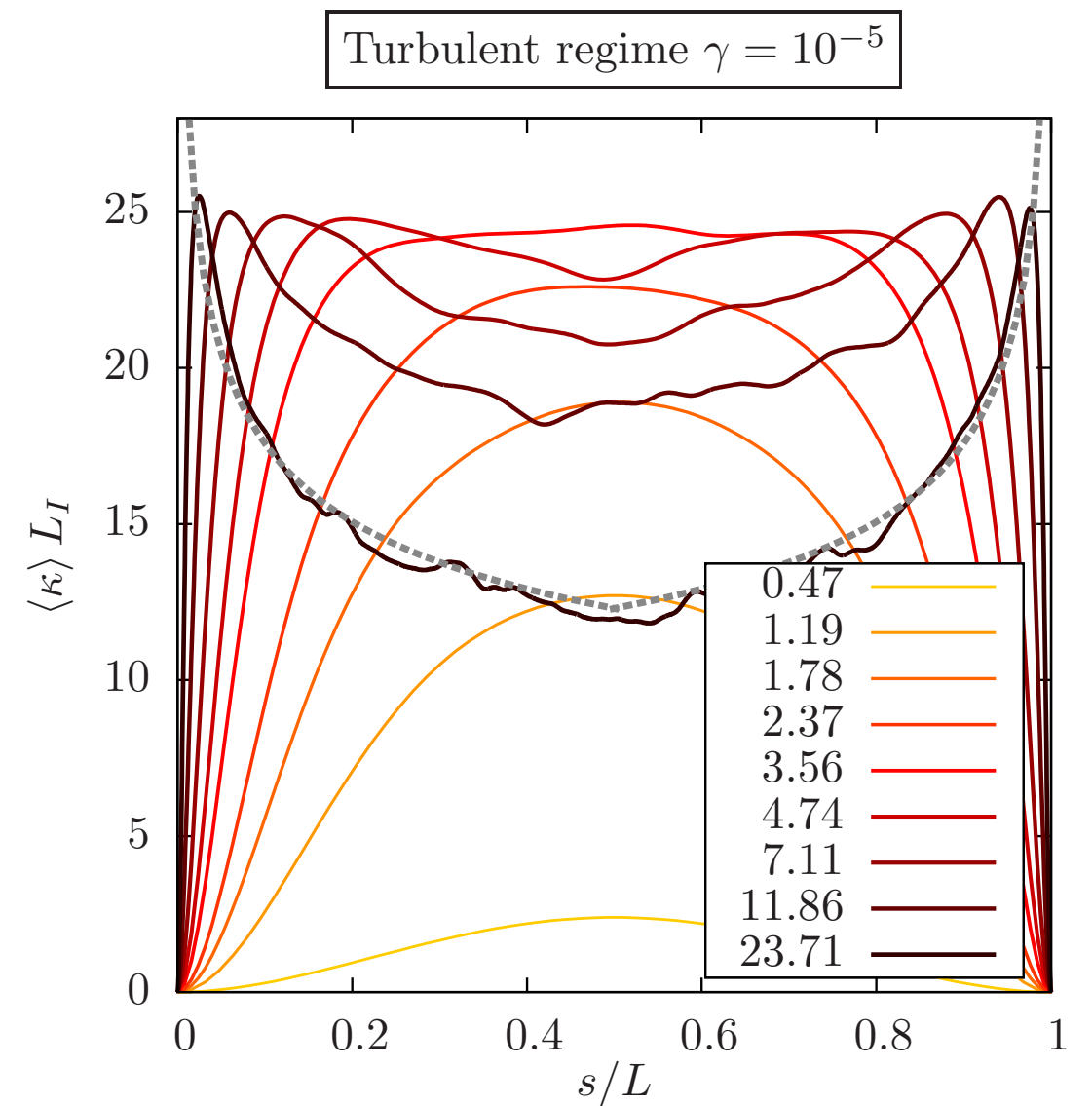


Turbulent regime

Spatial evolution of the local curvature



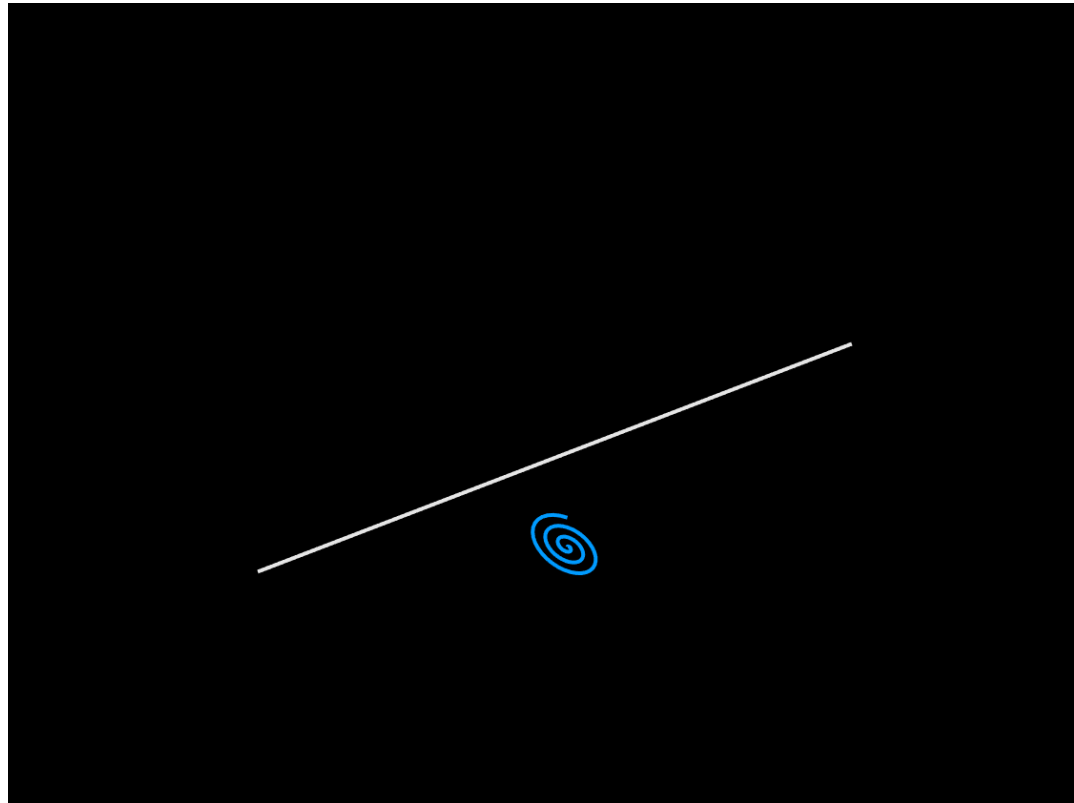
Polymer regime



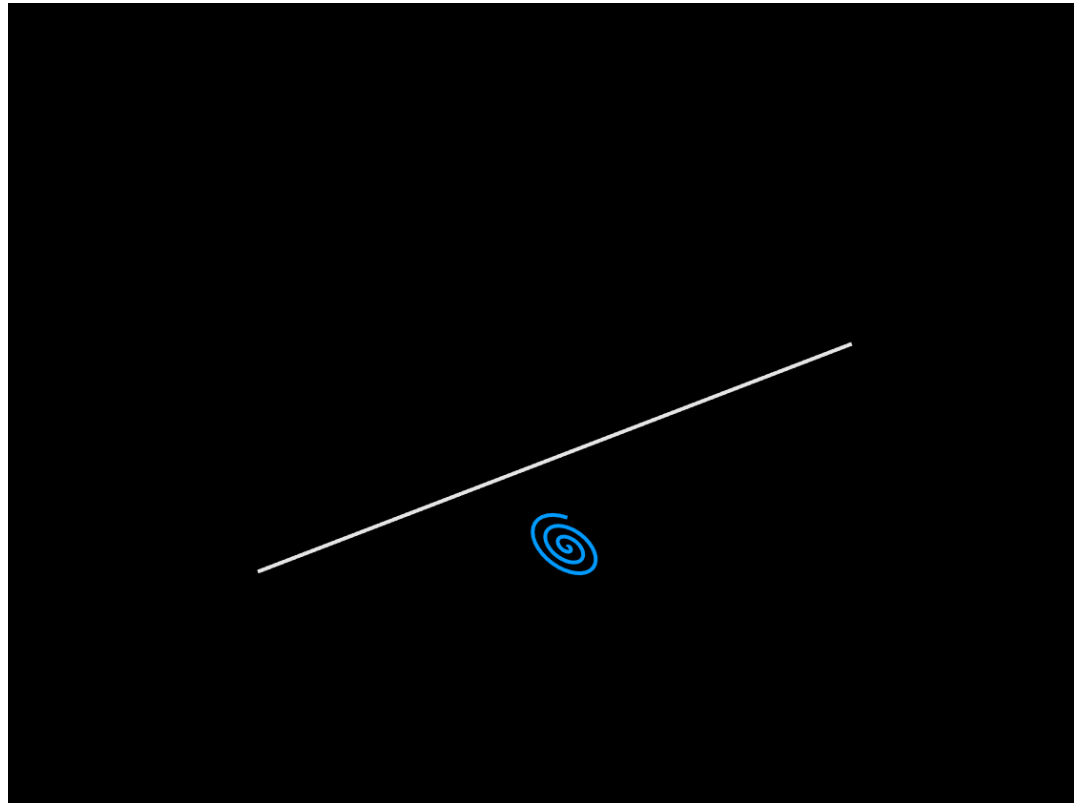
Turbulent regime

Straightening of long fibers in turbulence

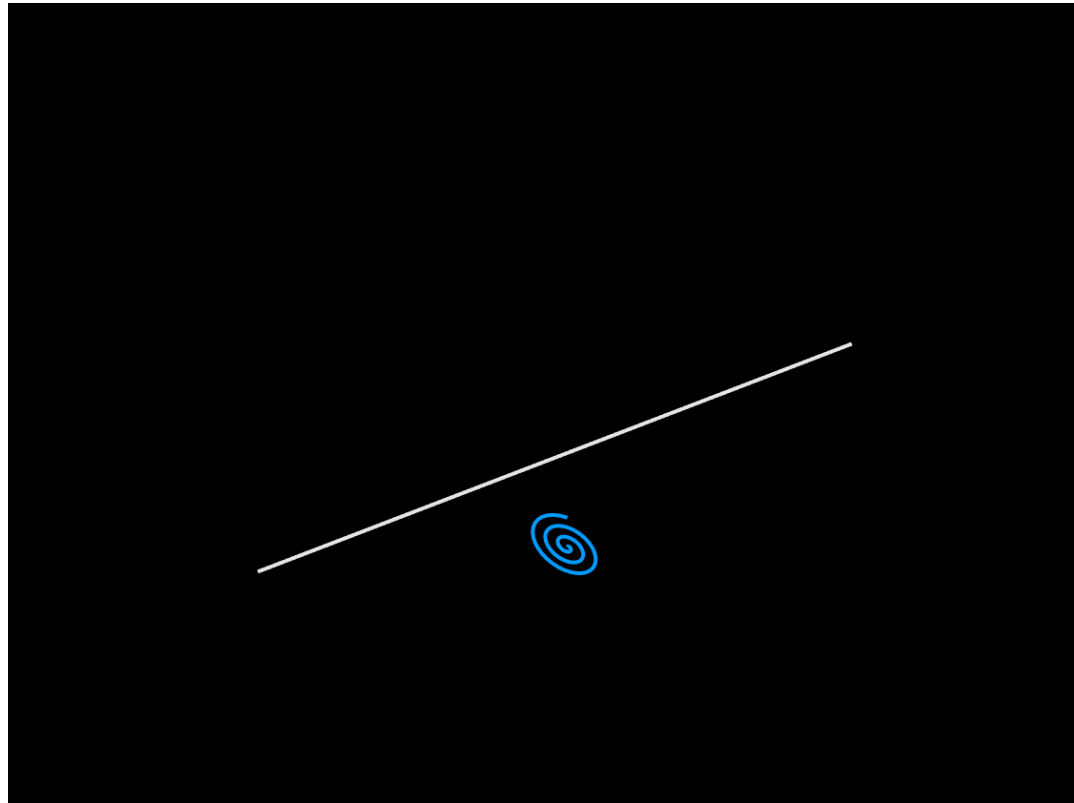
Spatial evolution of the local curvature



Spatial evolution of the local curvature

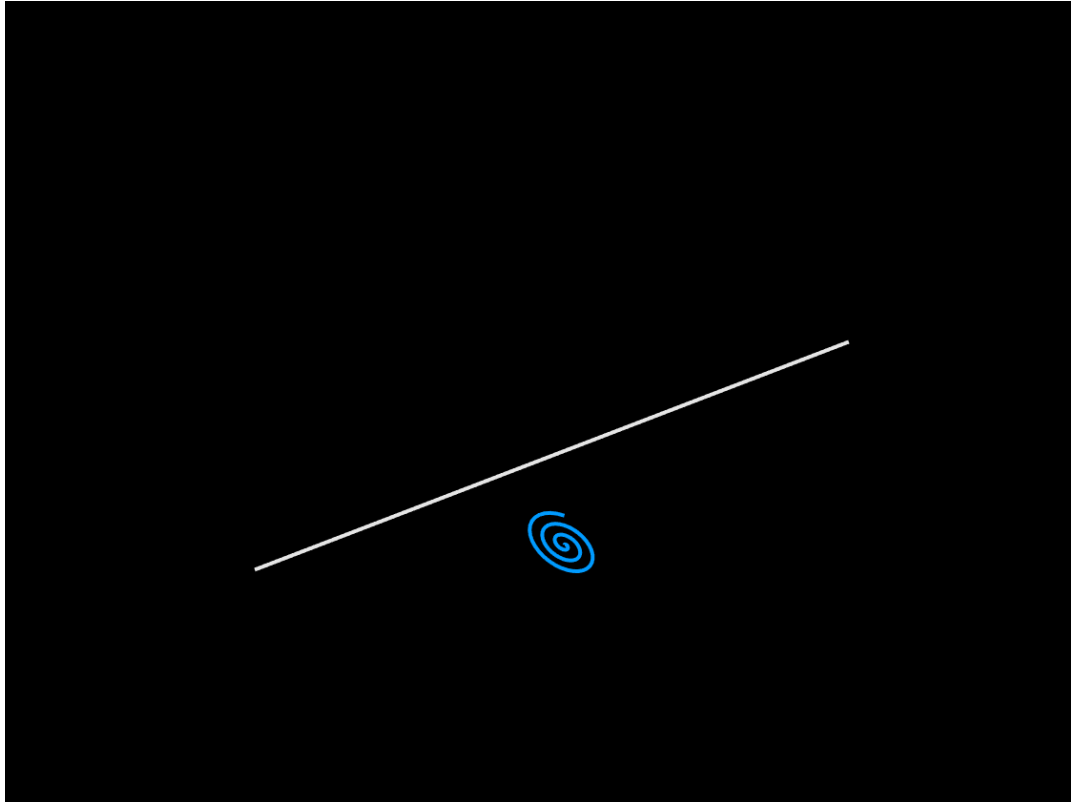


Spatial evolution of the local curvature



Local deformation => global modification

Spatial evolution of the local curvature



Local deformation => global modification

Additional dissipation term

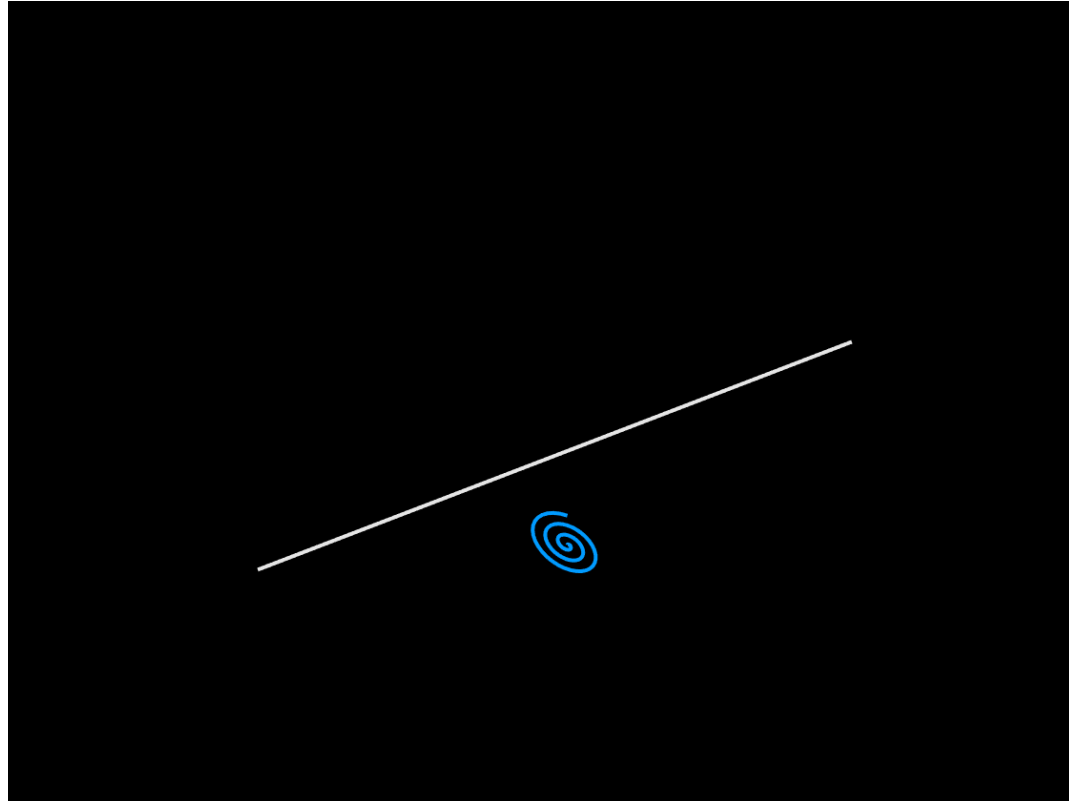
$$\mathcal{P}_v = \int \eta \partial_t \mathbf{r} \cdot \mathbf{u} ds$$

Fiber is inextensible

$$|\partial_t \mathbf{r}| \sim \frac{1}{\kappa \tau_\kappa} \quad \tau_\kappa \sim \frac{\eta}{EI \kappa^4}$$

$$\mathcal{P}_v \sim \frac{\eta}{\kappa \tau_\kappa} \int u_{\parallel} ds$$

Spatial evolution of the local curvature



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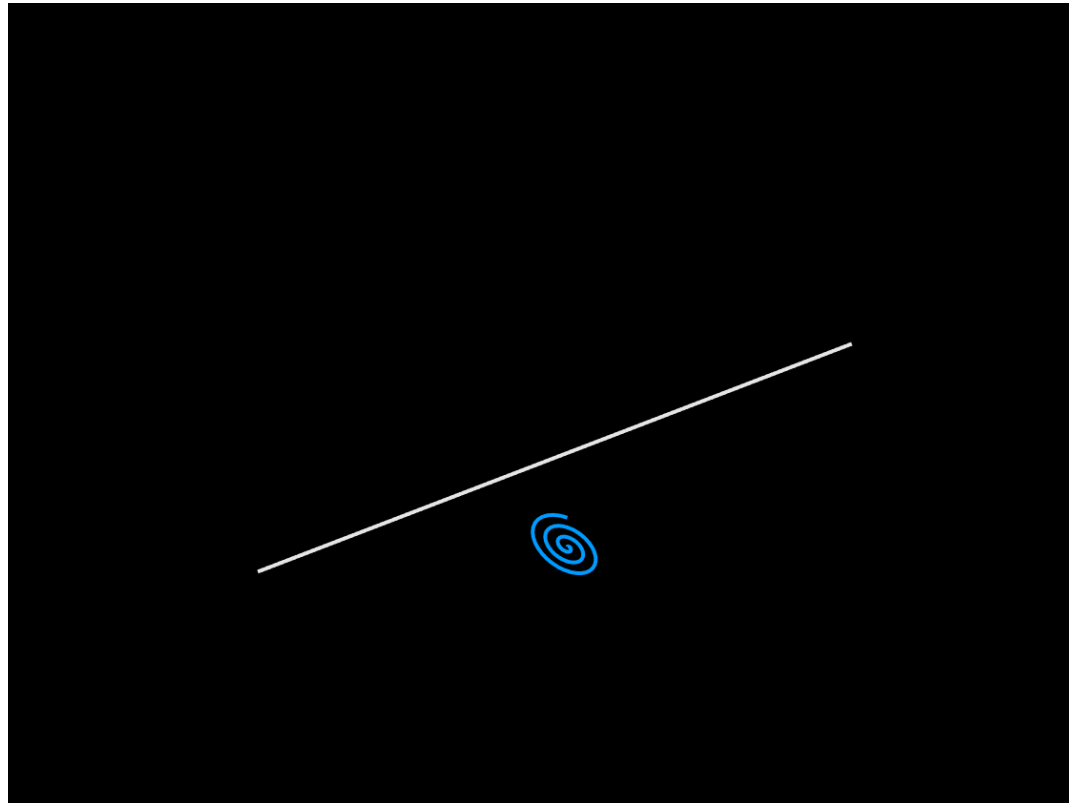
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Spatial correlation

$$\int_0^{s_0} u_{\parallel} ds \sim s_0 u_{s_0} \sim \epsilon^{1/3} s_0^{4/3}$$

Spatial evolution of the local curvature



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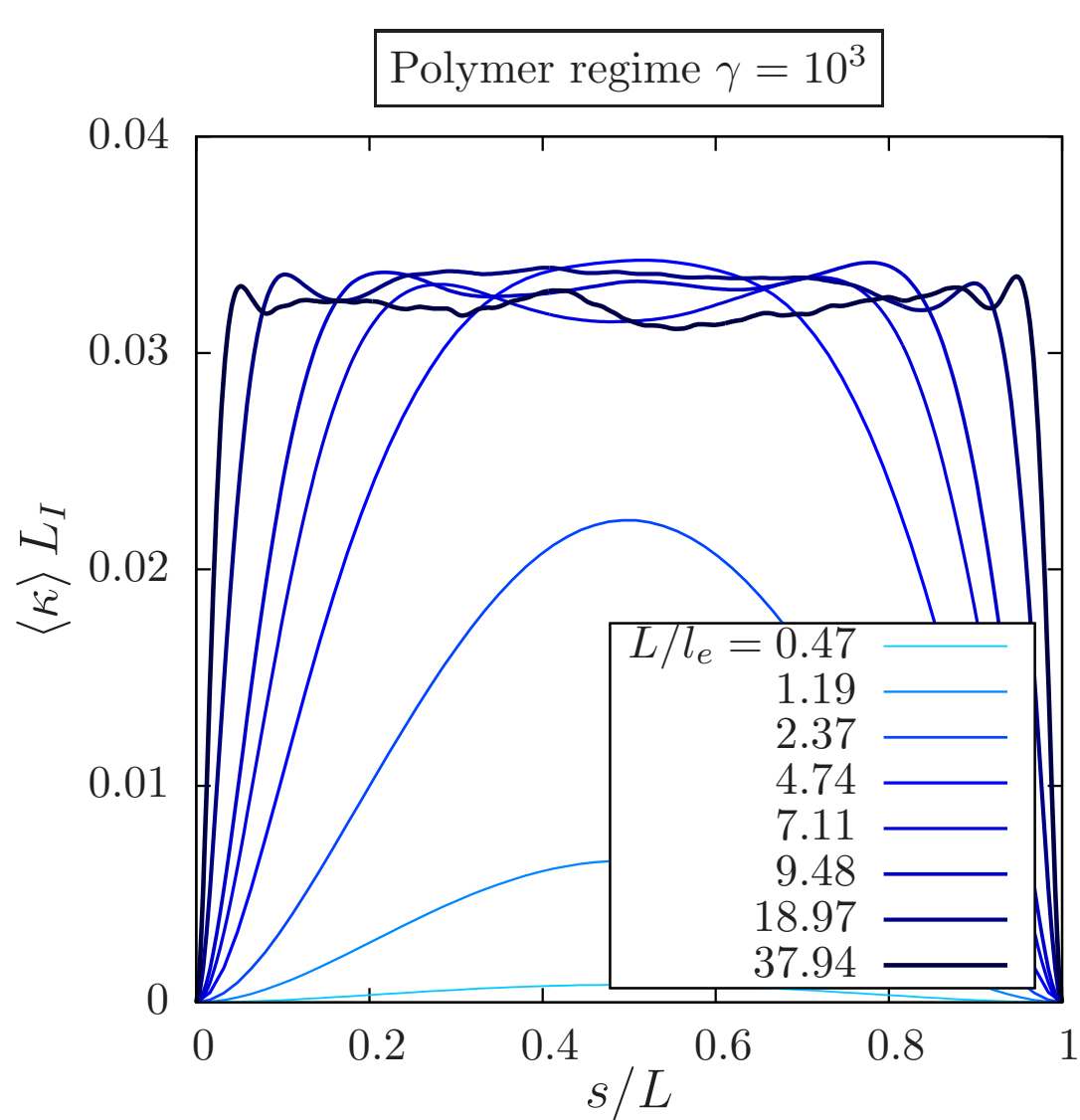
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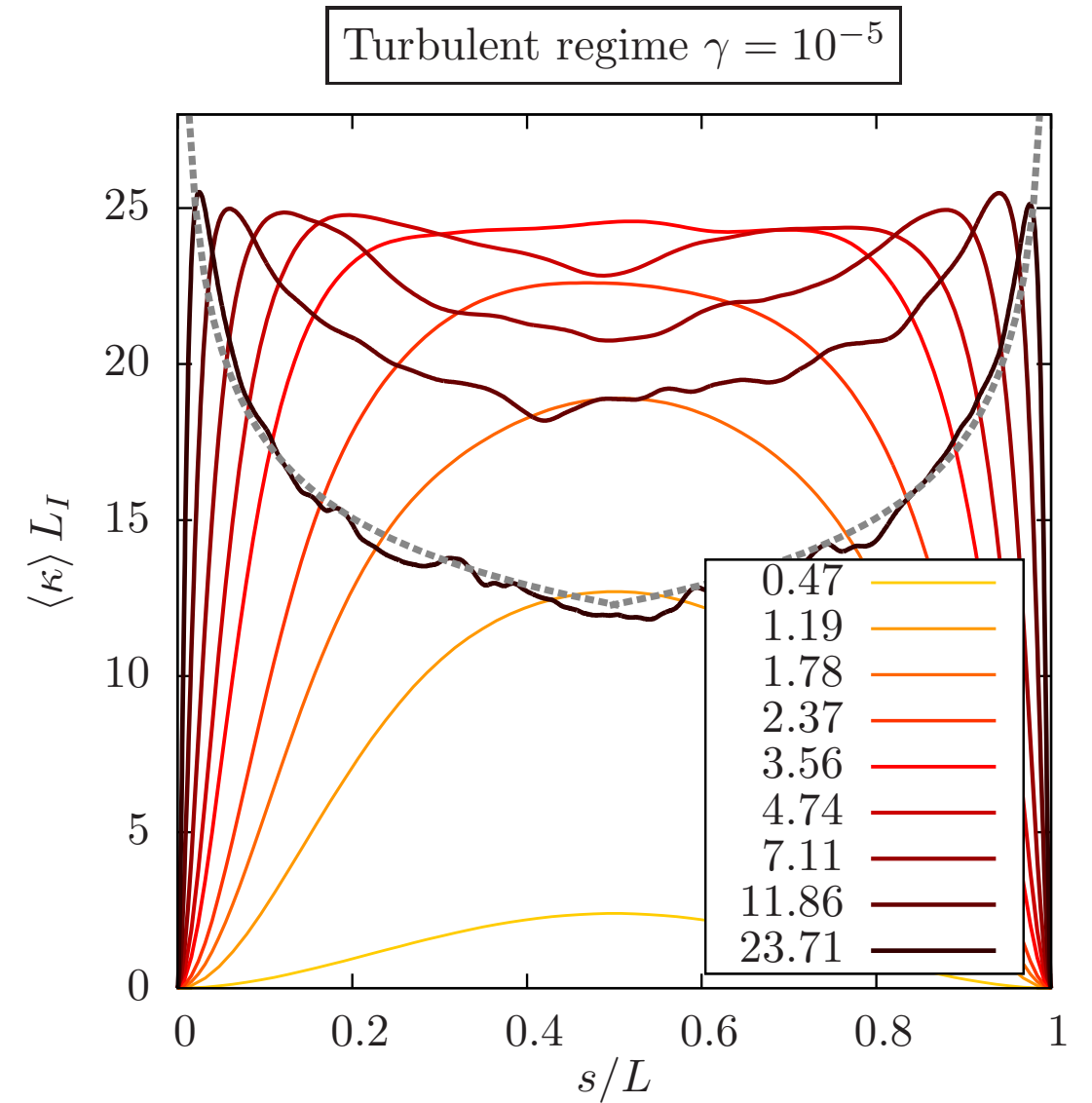
For long fiber $\mathcal{P}_{turb} = \rho \kappa^{-3} \epsilon \sim \mathcal{P}_v$

$$\kappa(s_0) \sim \left(\frac{\rho}{EI} \right)^{1/6} \epsilon^{1/9} s_0^{-2/9}$$

Spatial evolution of the local curvature



Polymer regime



turbulent regime

$$\kappa(s_0) \sim \left(\frac{\rho}{EI} \right)^{1/6} \epsilon^{1/9} s_0^{-2/9}$$

Conclusion on deformation of fibers

- Fiber longer than $\ell_e = (EI)^{1/4}/(\rho\eta\varepsilon)^{1/8}$ can be bent
- Straightening of the longest fibers due to flow correlation

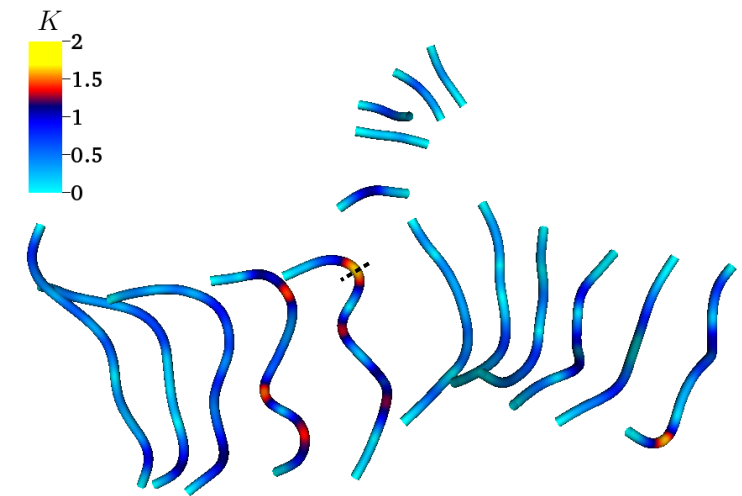
From deformation to fragmentation

Simplest approach: brittle object

No ageing (nor chemical nor mechanical)

Brittle object breaks if $\epsilon_{def} > \epsilon_{max}$

Deformation = strain $\epsilon_{def} = \kappa d$



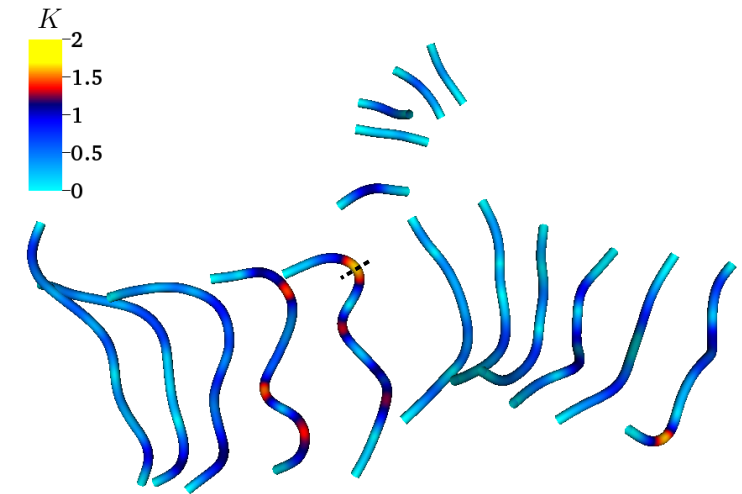
Independent breaking event

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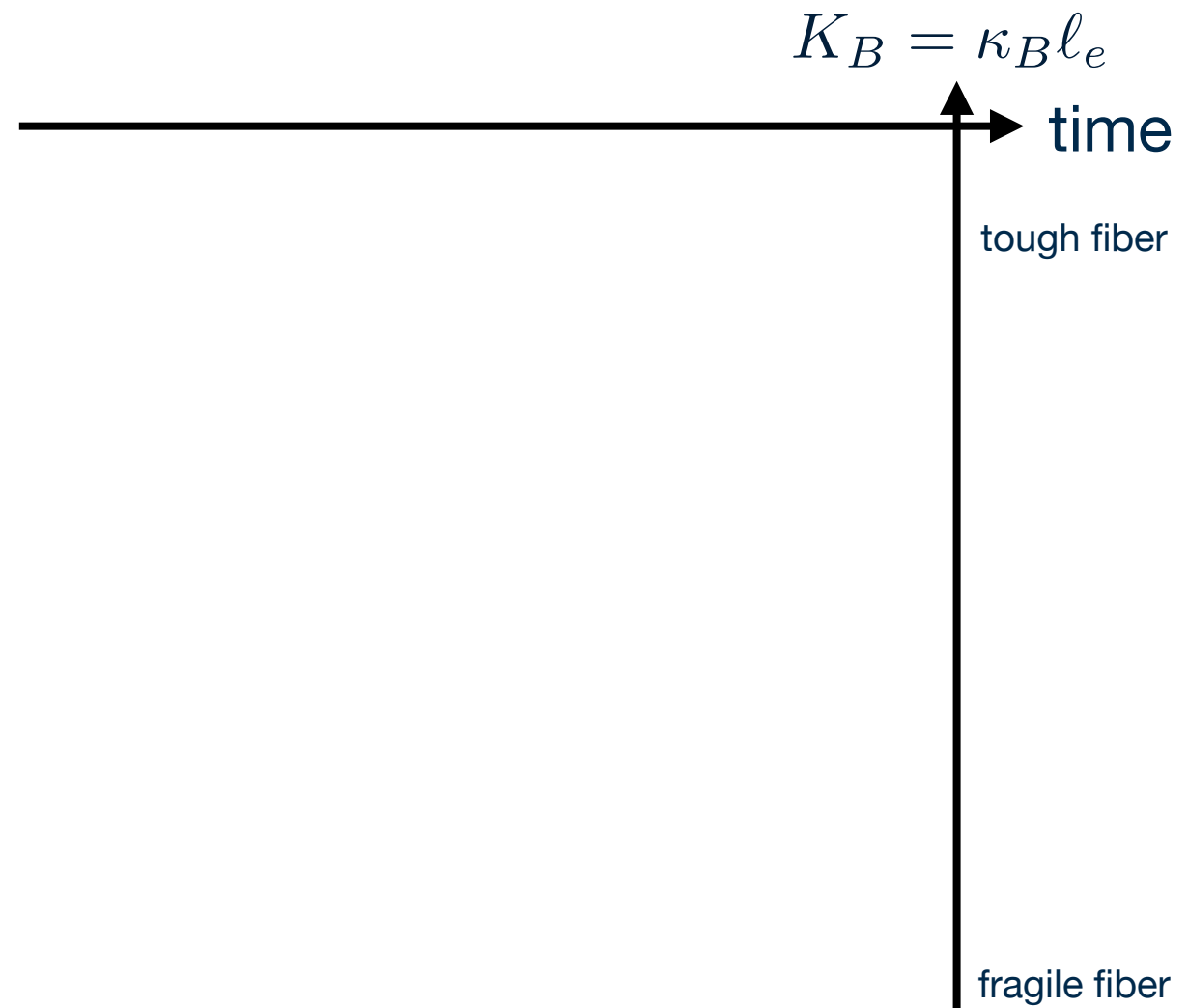


Independent breaking event

Two parameters : maximum curvature κ_B + duration of the experiments

Simplest approach

Experiment with glass fibers

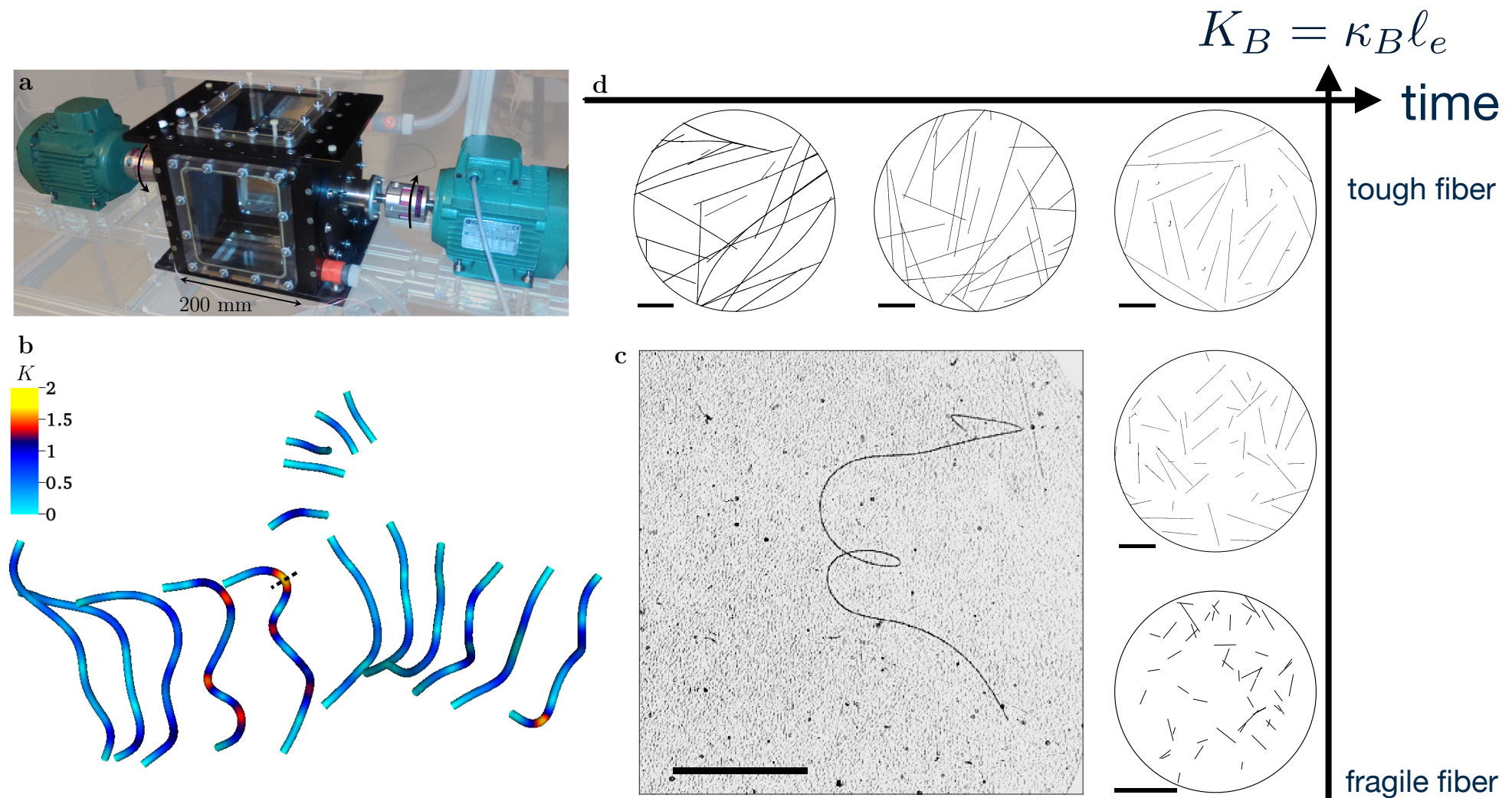


2 different fluids: water / water+Ucon (change viscosity) $\ell_e = \frac{(EI)^{1/4}}{(\rho\eta\varepsilon)^{1/8}}$

raw fiber or heated fiber: modification of κ_B

Simplest approach

Experiment with glass fibers

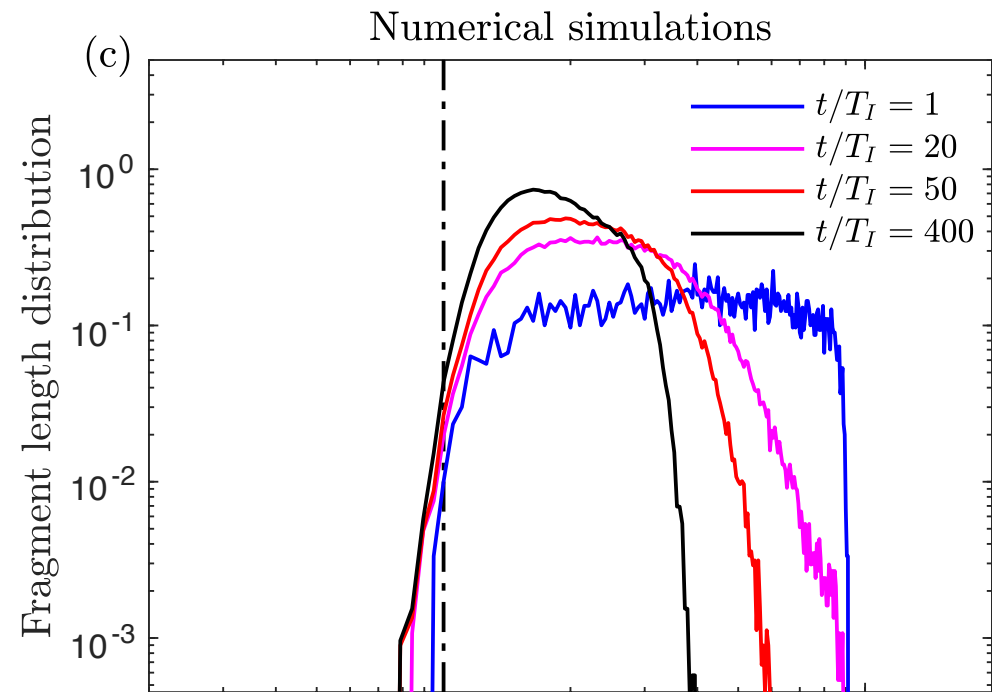
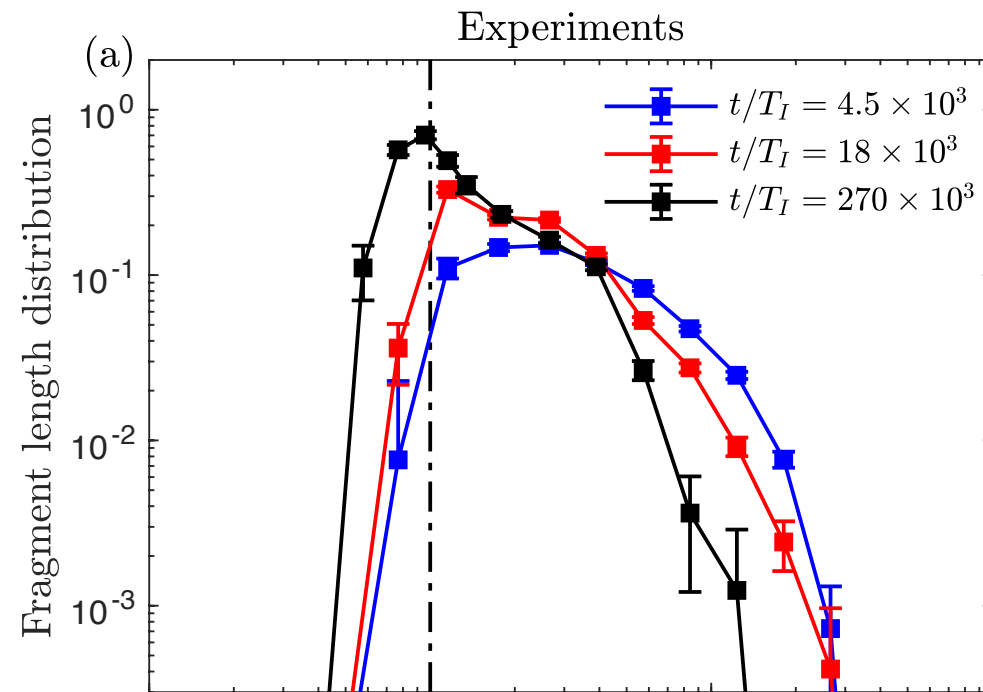


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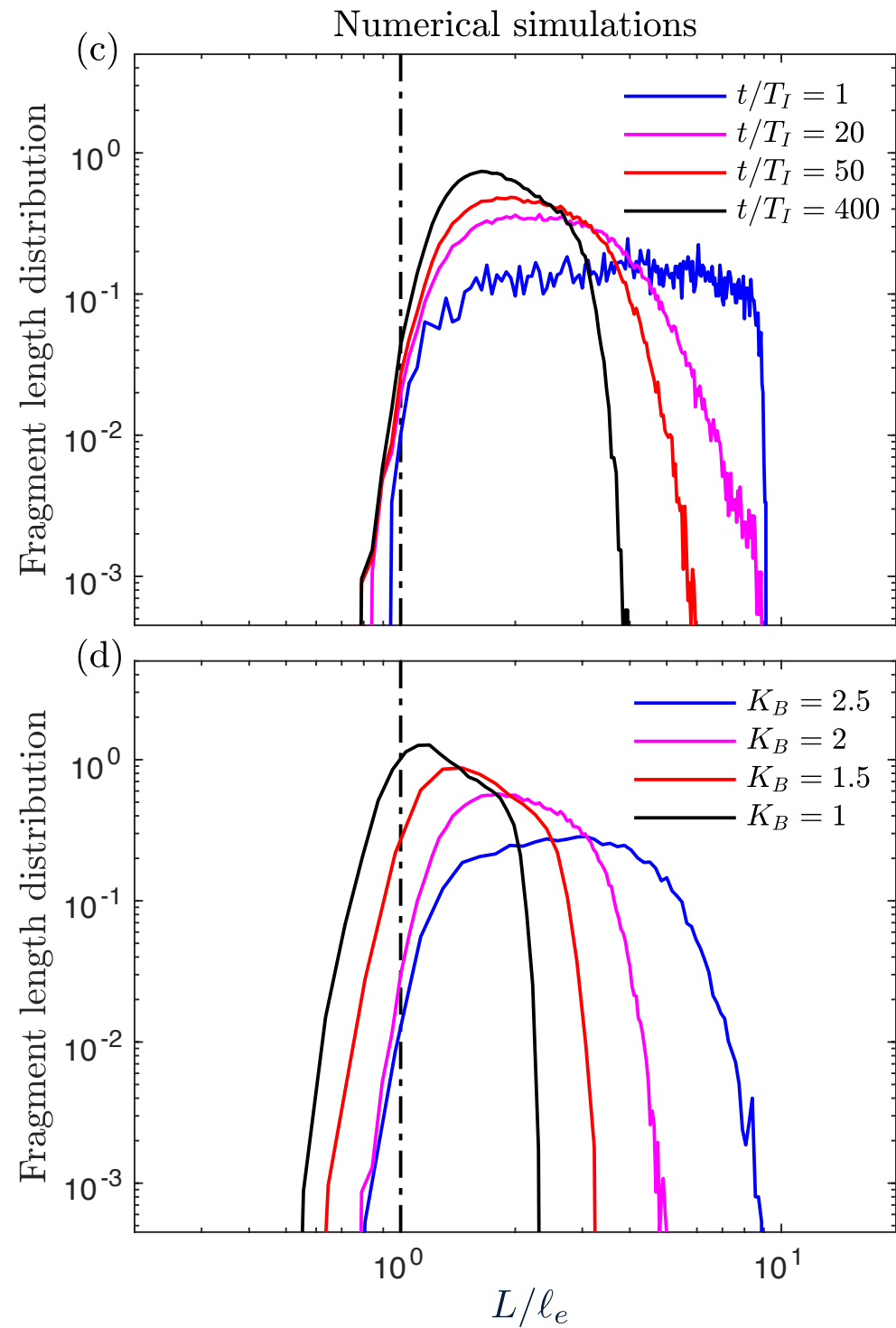
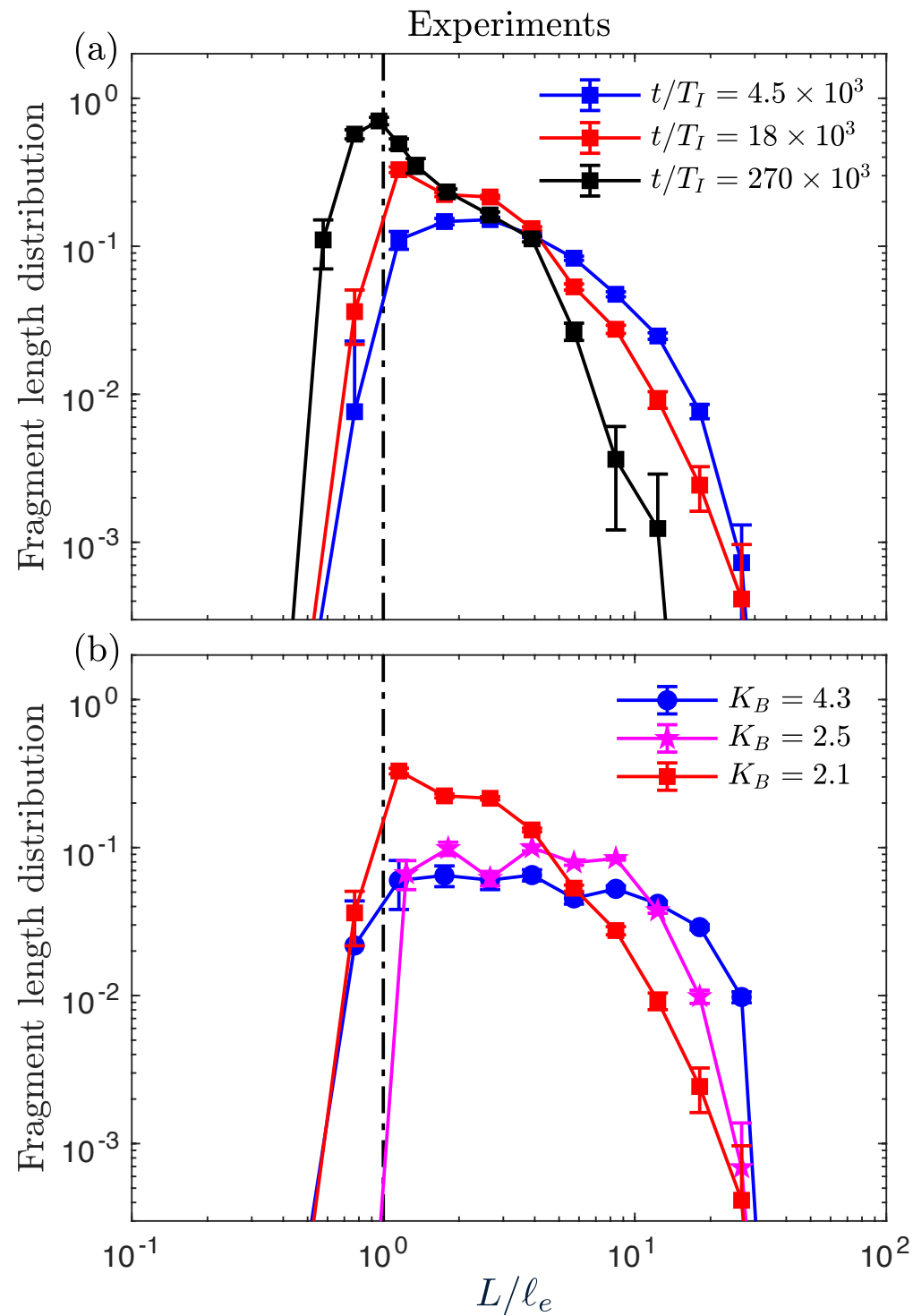
Evolution of the fragment size distribution

Time dependence



Evolution of the fragment size distribution

Time dependence



Modeling the evolution of the fragment size distribution

$n_L(t)$ number of fiber of size L at time t

$$\partial_t n_L = -n_L p(L) + 2 \int_L^\infty n_{L'} p(L') \gamma(L, L') dL'$$

$p(L)$ probability of breaking per unit time for a fiber of length L

$\gamma(L, L')$ probability that a fiber of length L' breaks into a fragment of size L

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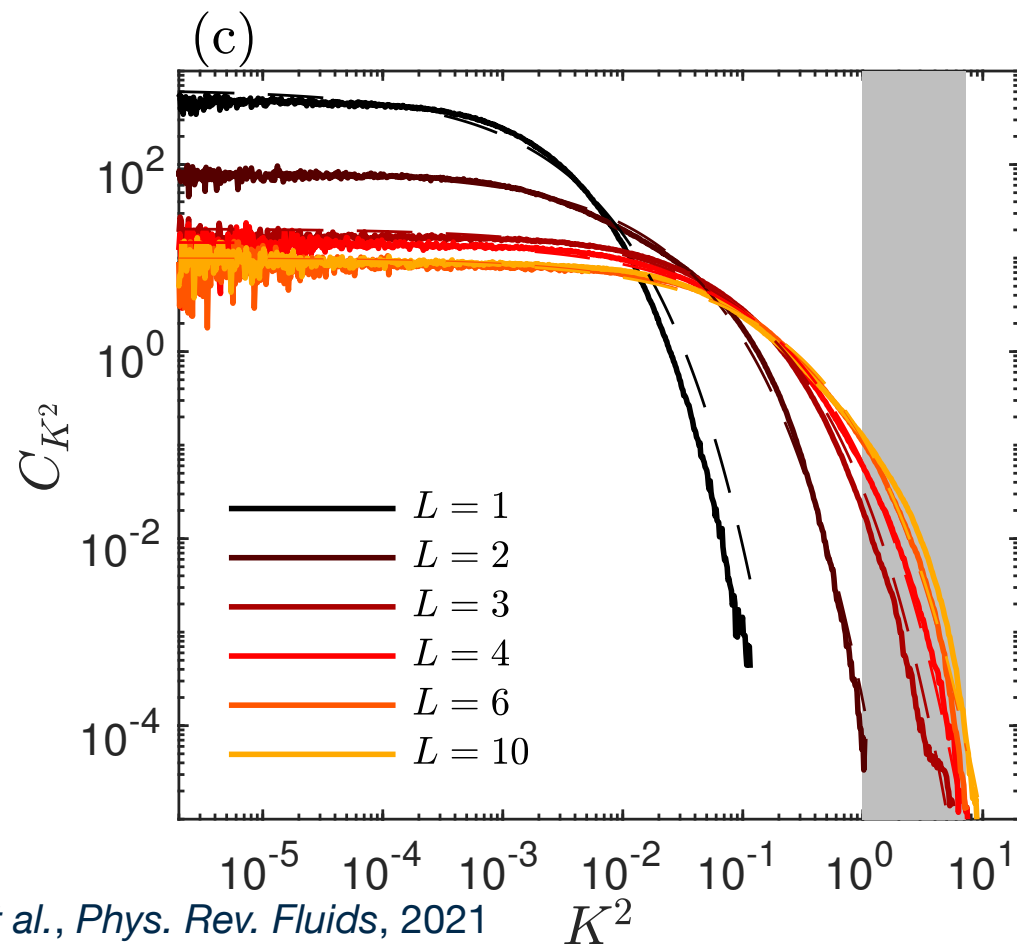
Modeling the evolution of the fragment size distribution

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$p(L)$ probability of breaking per unit time for a fiber of length L

Depends on the statistics of the local curvature



Probability to break locally

$$p_L(K_B) = \int_{K_B^2}^\infty C_{K^2}(x) dx$$

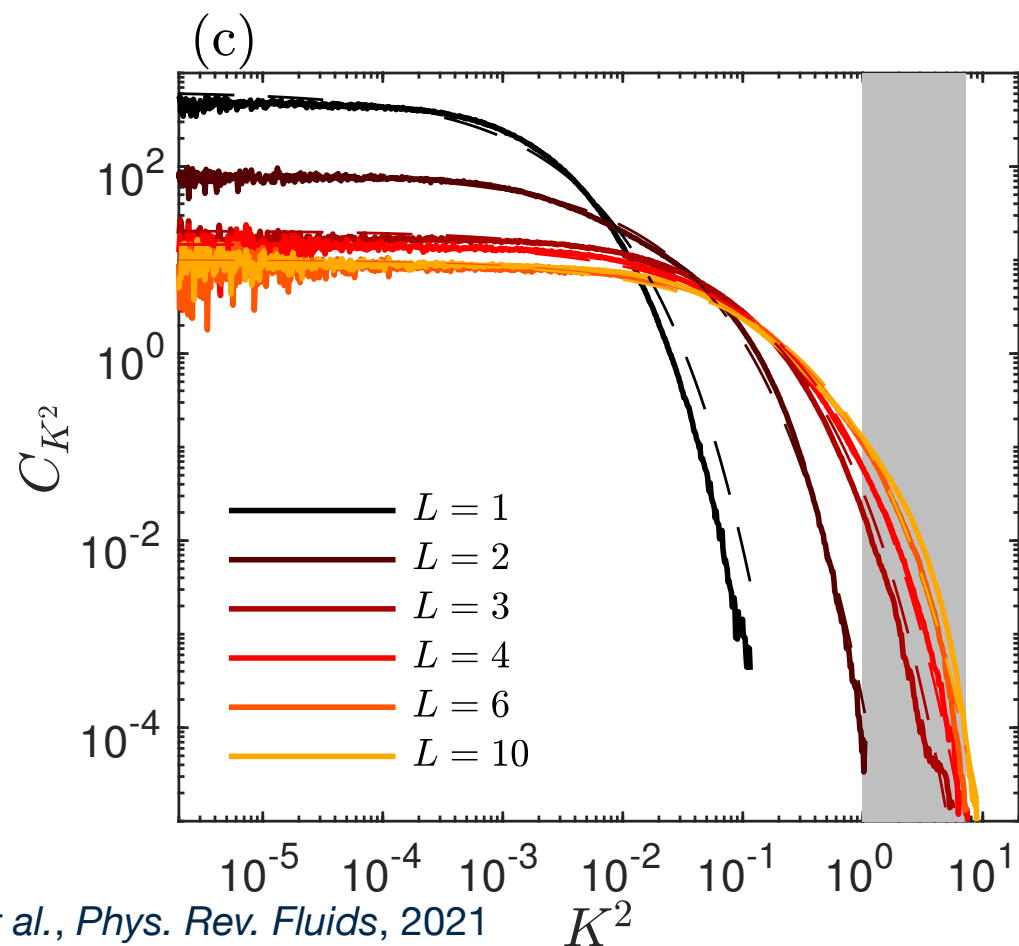
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Probability to break locally

$$p_L(K_B) = \int_{K_B^2}^\infty C_{K^2}(x) dx$$

Probability that a fiber breaks

$$p(L) = 1 - (1 - p_L(K_B))^{L/\ell_e}$$

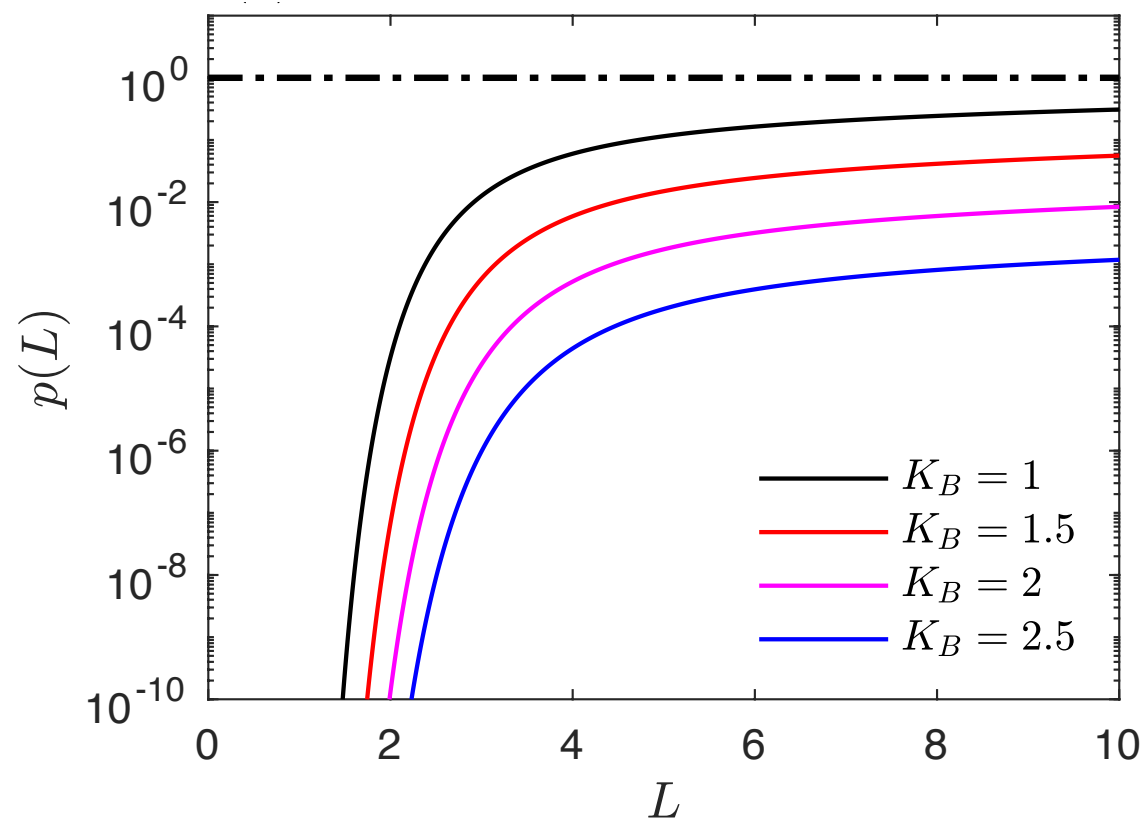
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Modeling the evolution of the fragment size distribution

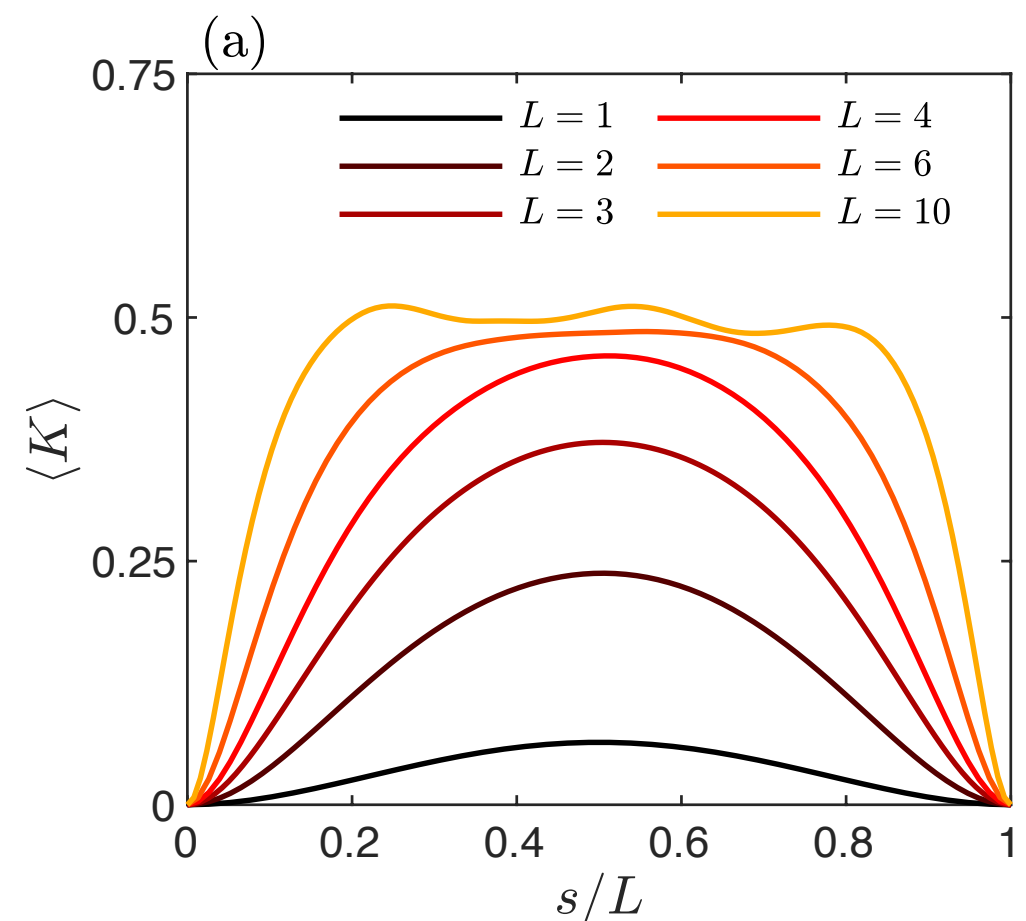
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For long fibers: plateau at the center

Boundary condition $\kappa(s = 0, L) = 0$



Modeling the evolution of the fragment size distribution

$n_L(t)$ number of fiber of size L at time t

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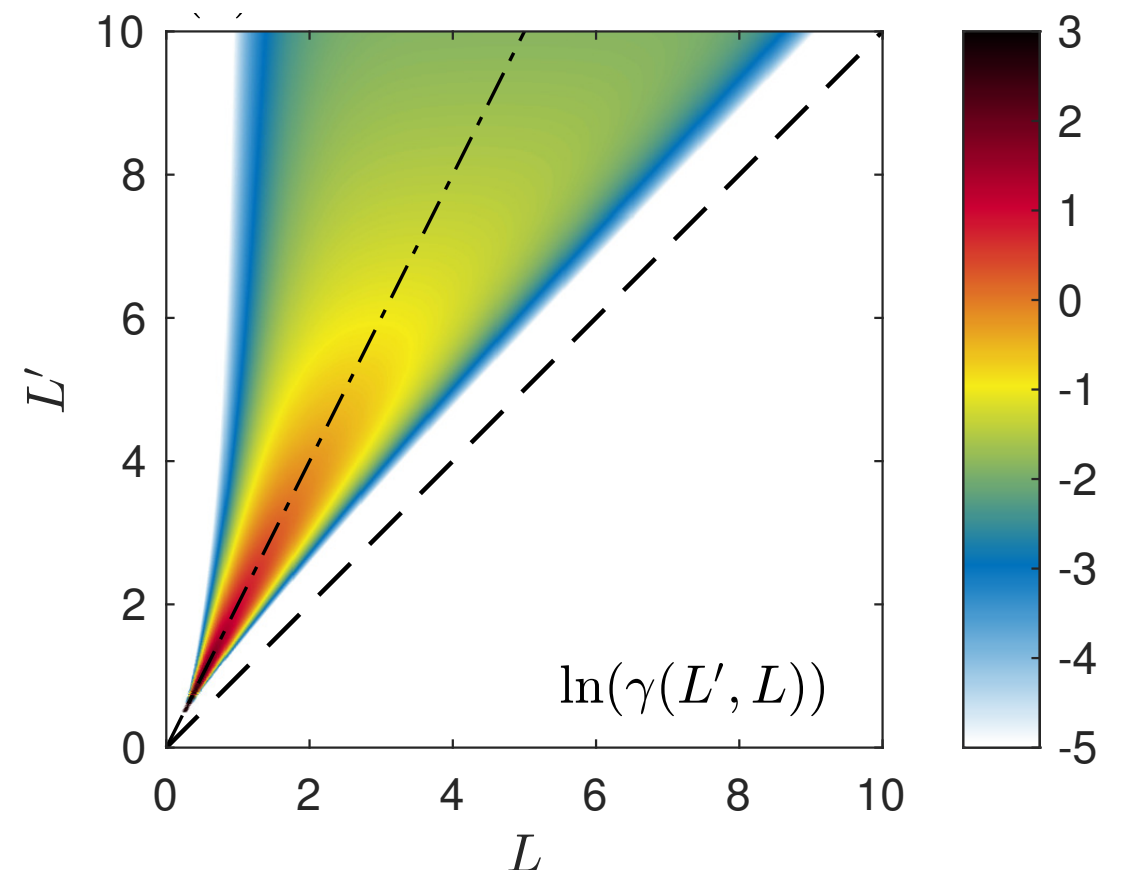
For long fibers: plateau at the center

Boundary condition $\kappa(s = 0, L) = 0$

short fibers break mainly at the center

Long fibers break uniformly

No breaking at the boundary



Comparison model/numerics

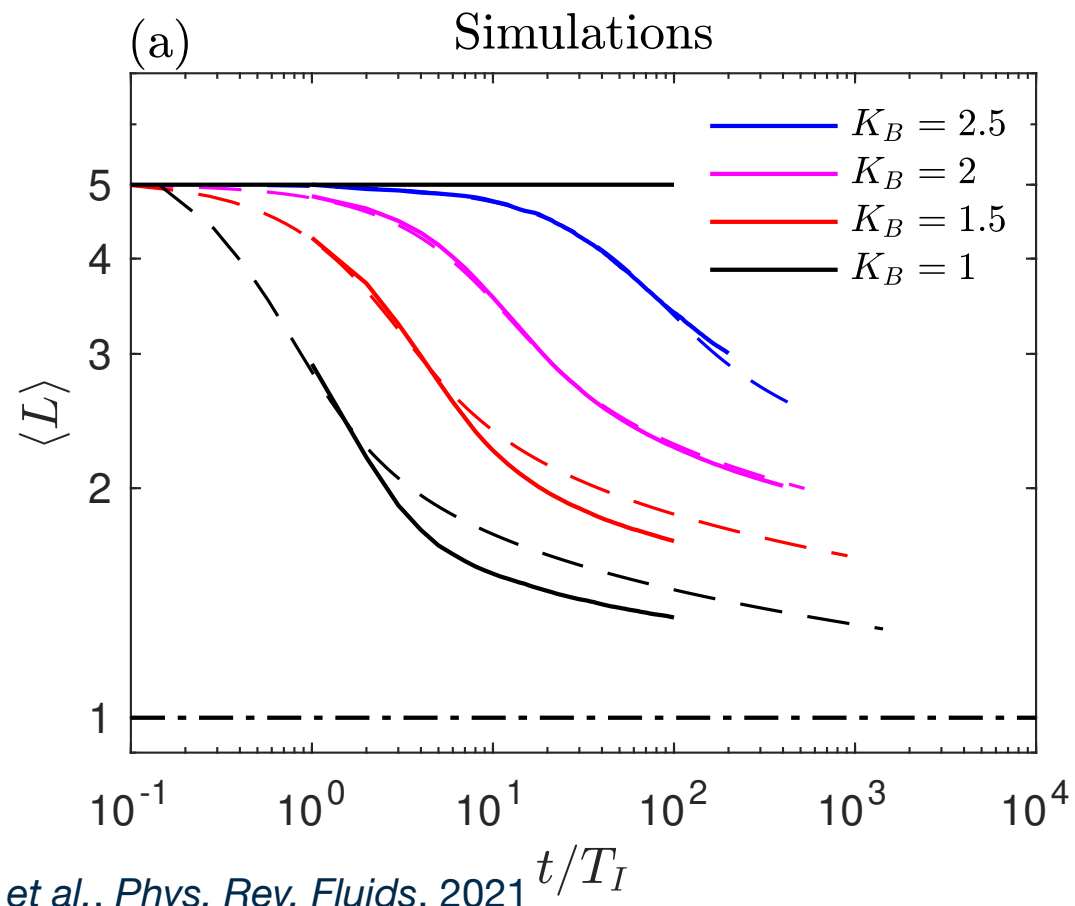
$$\partial_t n_L = -n_L p(L) + 2 \int_L^\infty n_{L'} p(L') \gamma(L, L') dL'$$

timescale determined by $\langle L \rangle$

Comparison model/numerics

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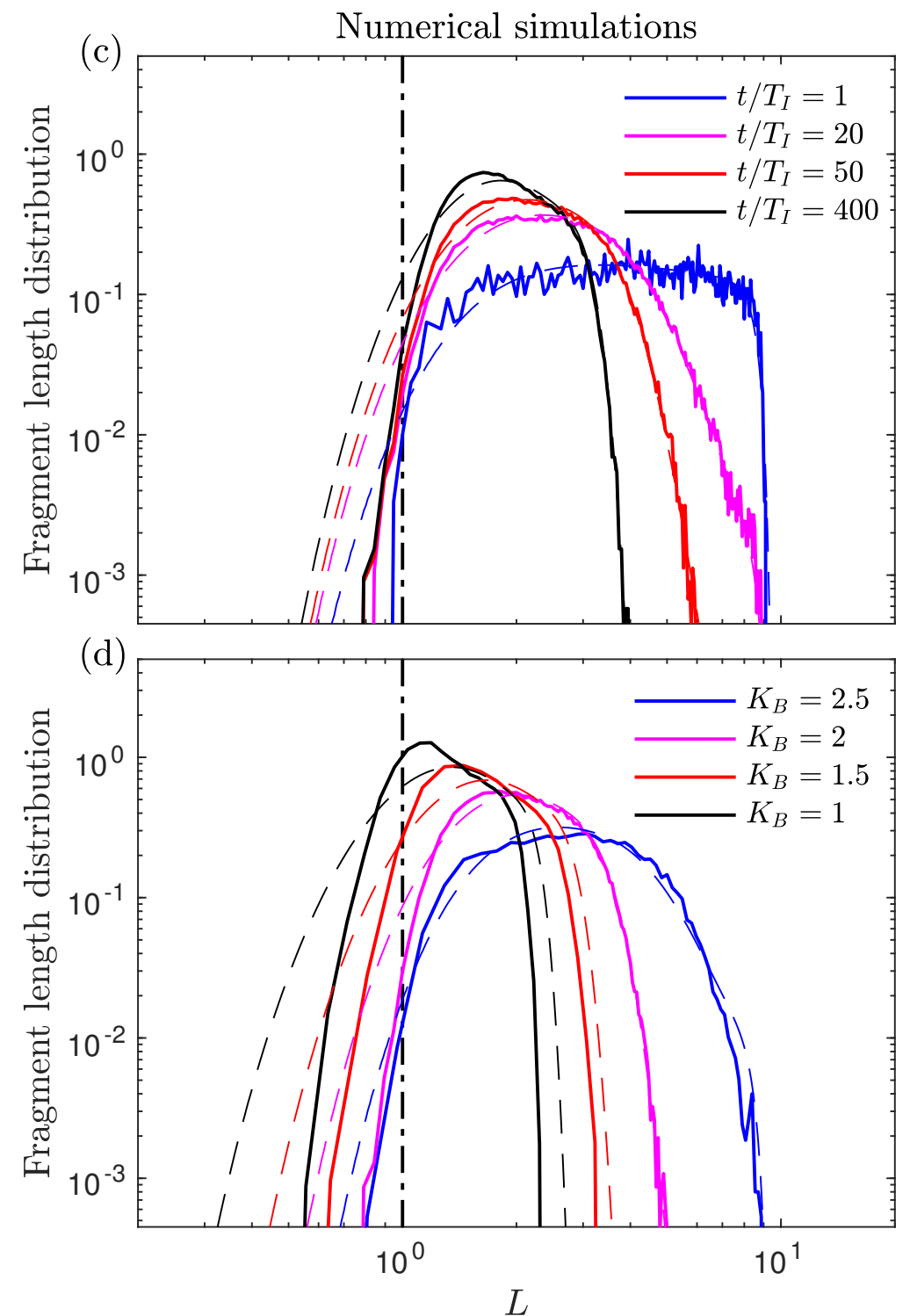
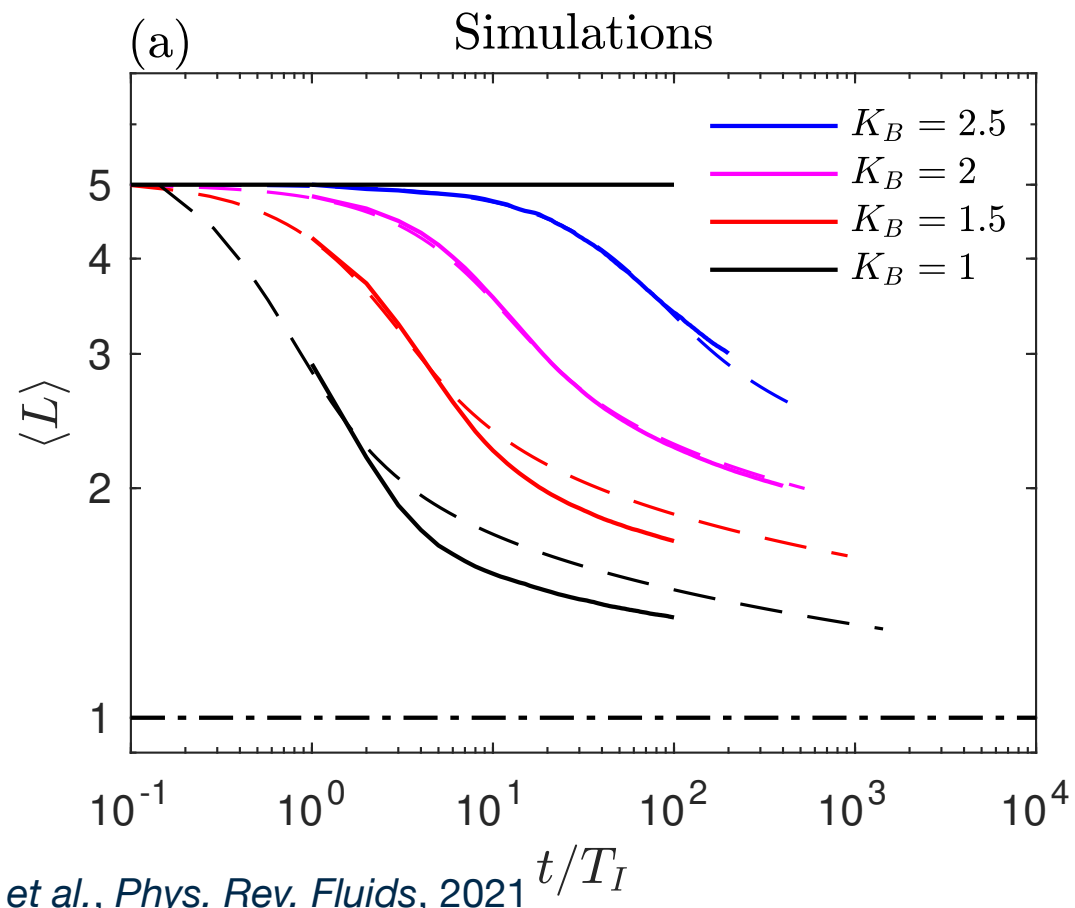
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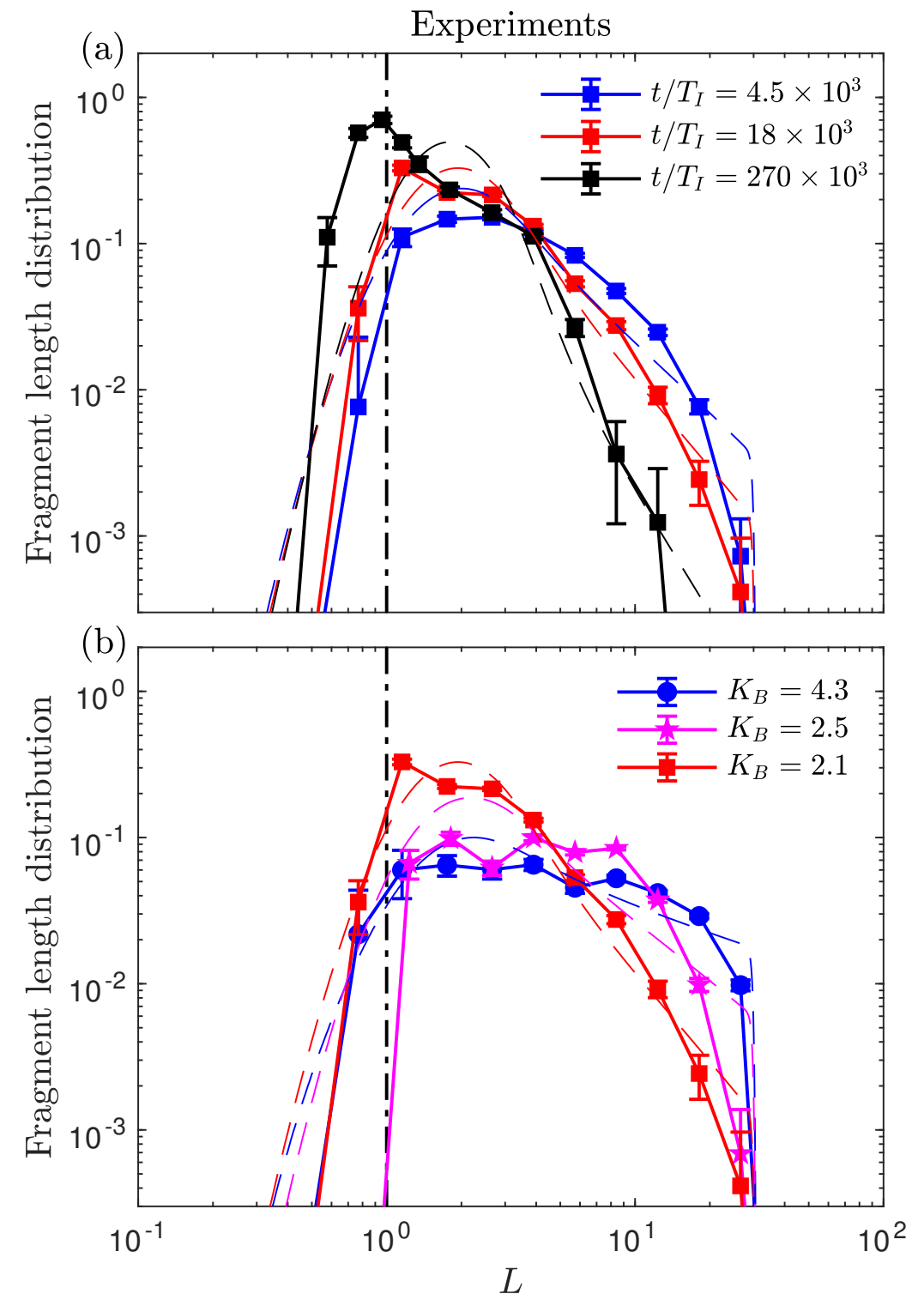
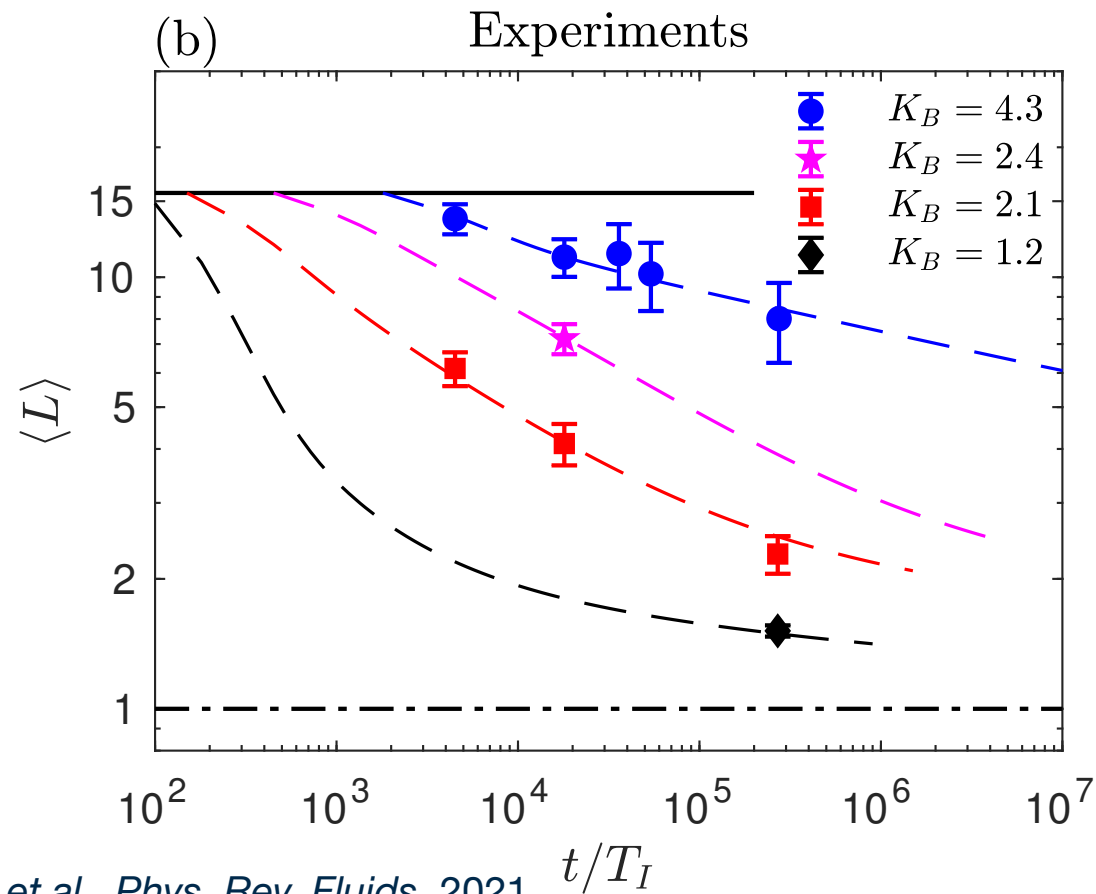


Comparison model/experiments

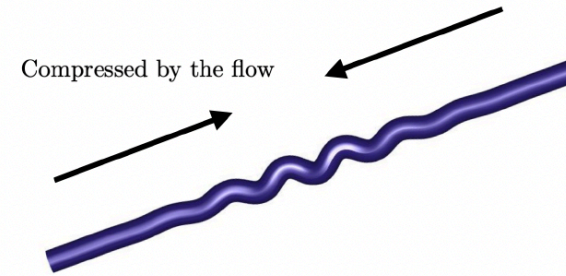
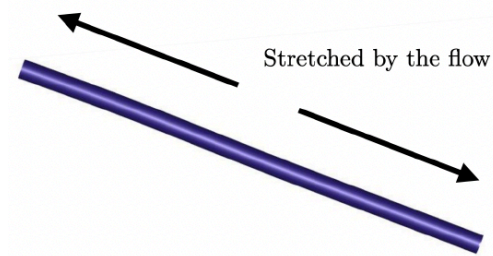
$$\partial_t n_L = -n_L p(L) + 2 \int_L^\infty n_{L'} p(L') \gamma(L, L') dL'$$

timescale determined by $\langle L \rangle$

Importance of defects (distribution of κ_B)

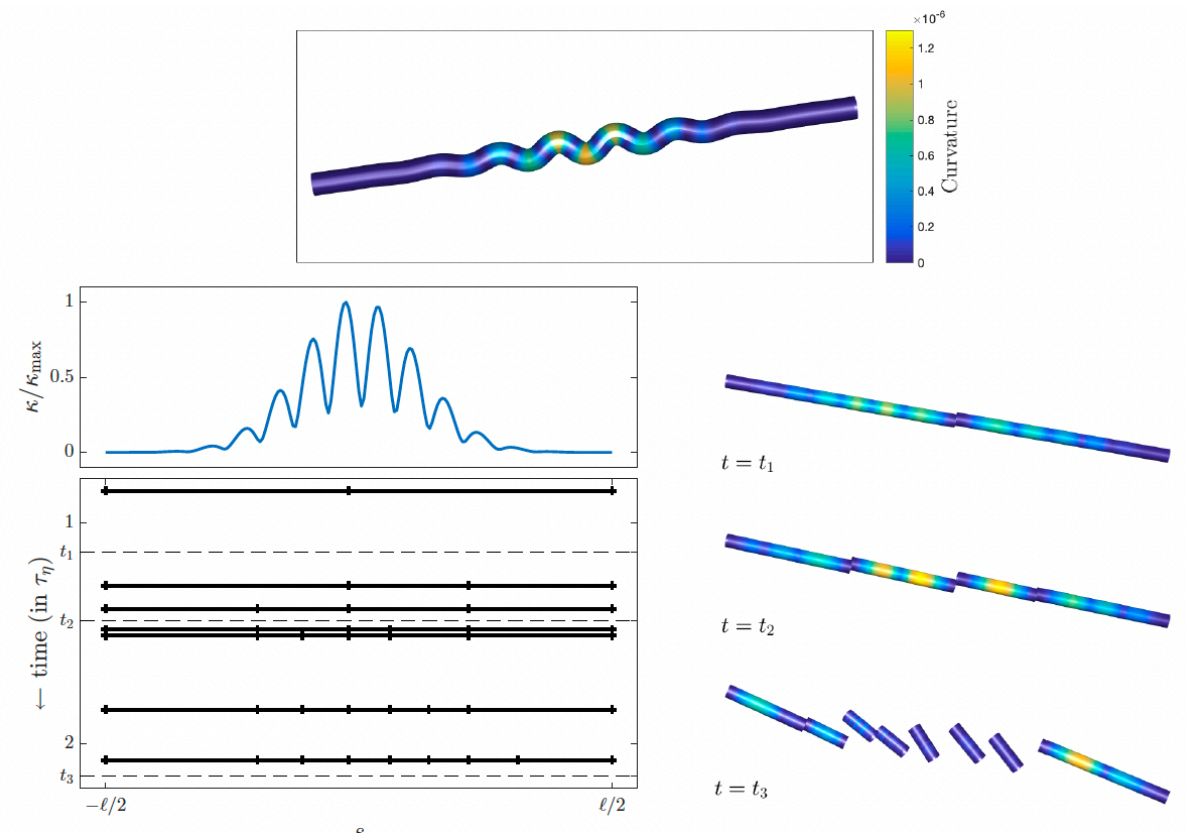
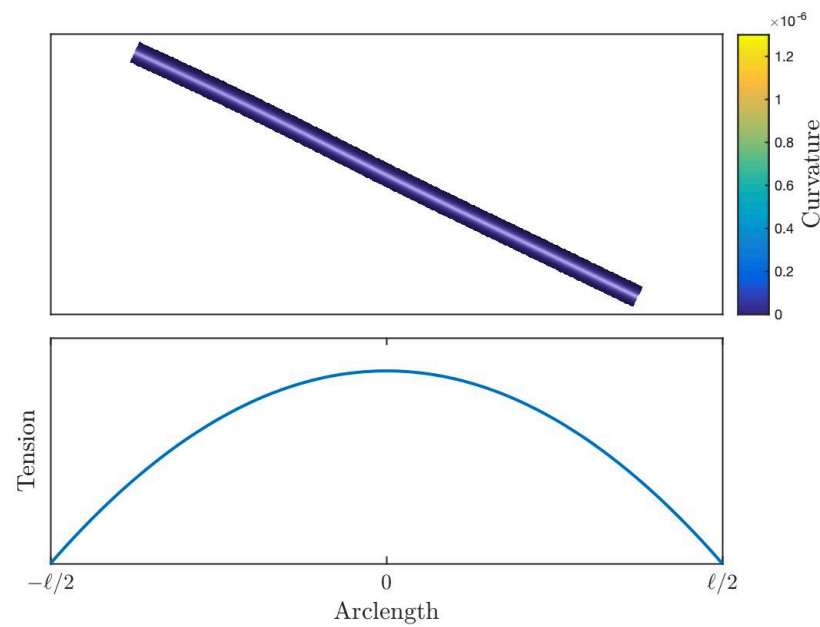


Fragmentation of small fibers



Fragmentation due to tension

Fragmentation due to buckling



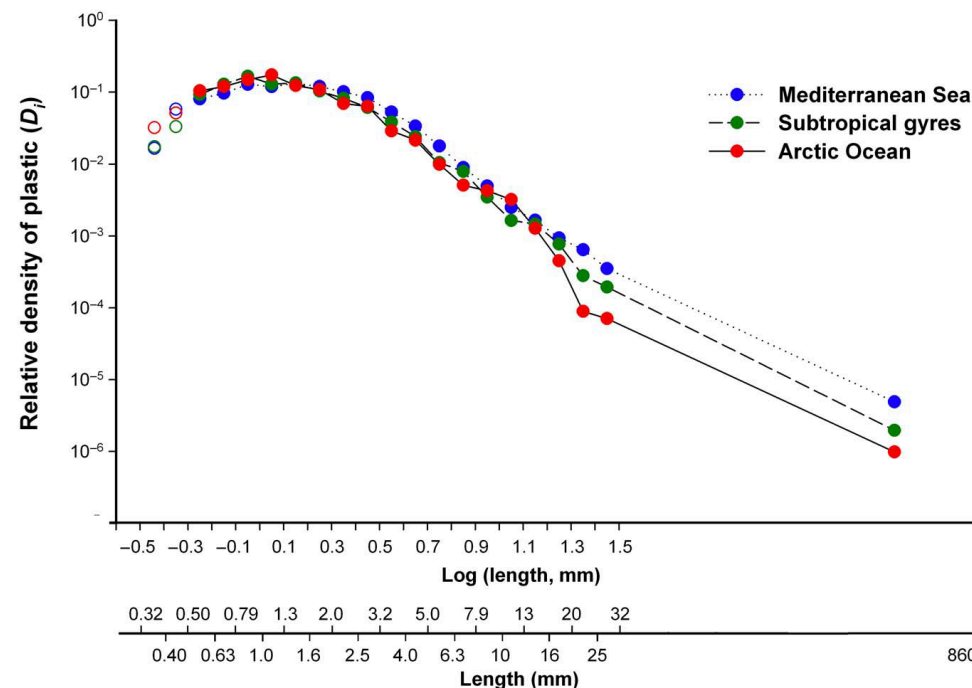
Possibility to form very small fragments

Conclusion on the fragmentation of fibers

- No new fragmentation mechanism at small scale are necessary to model the pdf
- Maximum of the pdf is given by the elastic length

Conclusion on the fragmentation of fibers

- No new fragmentation mechanism at small scale are necessary to model the pdf
- Maximum of the pdf is given by the elastic length



$$\ell_e \sim \frac{(EI)^{1/4}}{(\rho\eta\epsilon)^{1/8}}$$

$$E \sim 1 \text{ GPa} \quad d \sim 50 \text{ } \mu\text{m}$$

$$\epsilon \sim 10^{-1} - 10^2 \text{ m}^2 \cdot \text{s}^{-3}$$

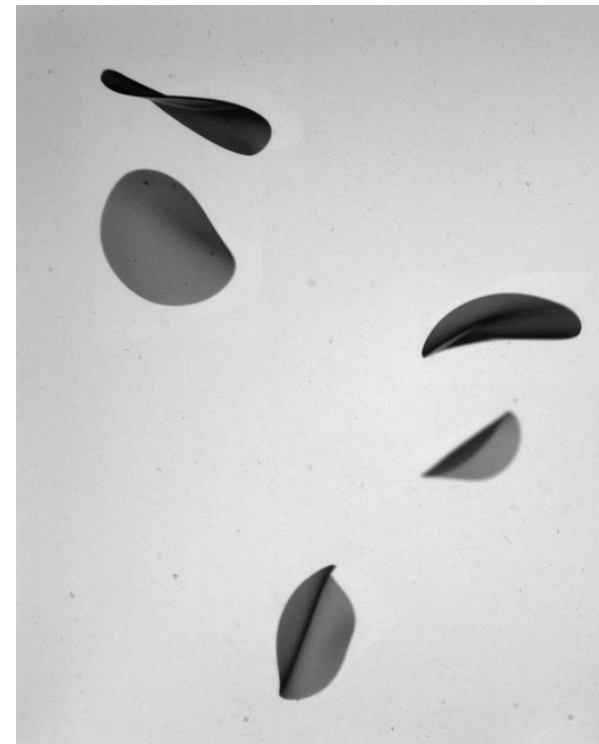
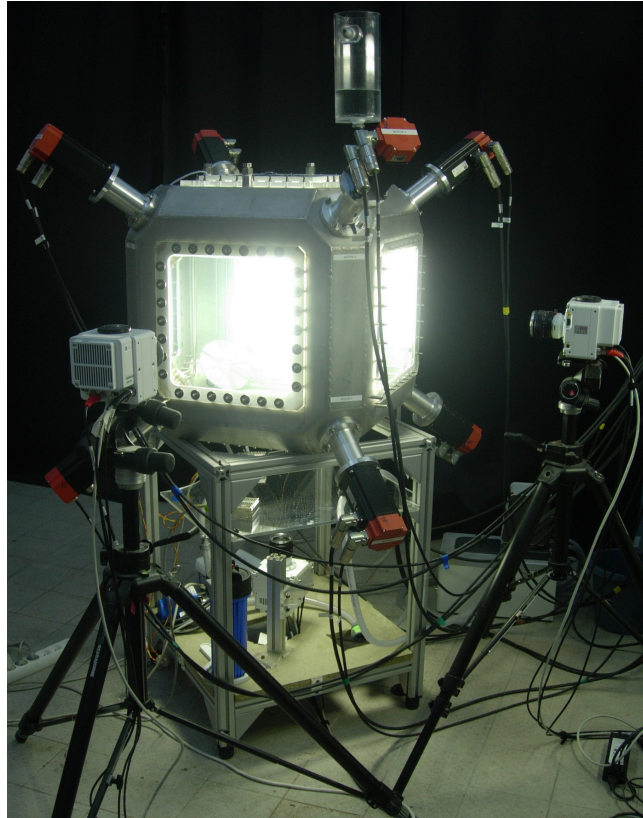
$$\ell_e \sim 2 - 5 \text{ mm}$$

On going and future work

- More complex shapes

Deformability of disc in turbulence

Deformability of disc in turbulence



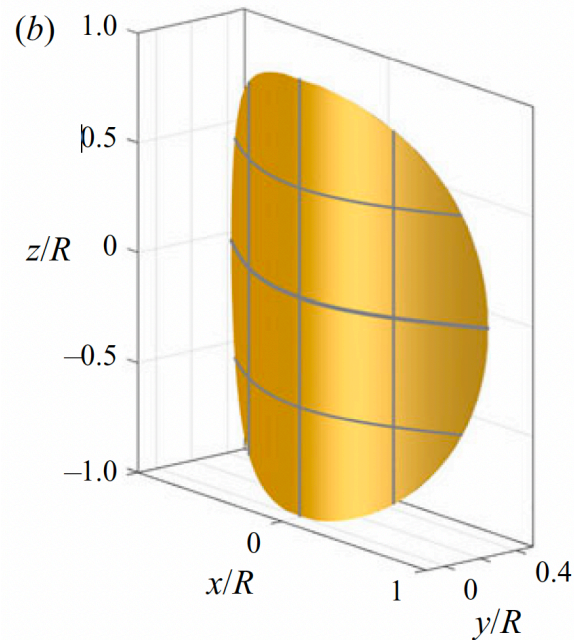
$$R_\lambda \in [300; 710]$$

$$\eta_K \in [30; 70] \mu\text{m}$$

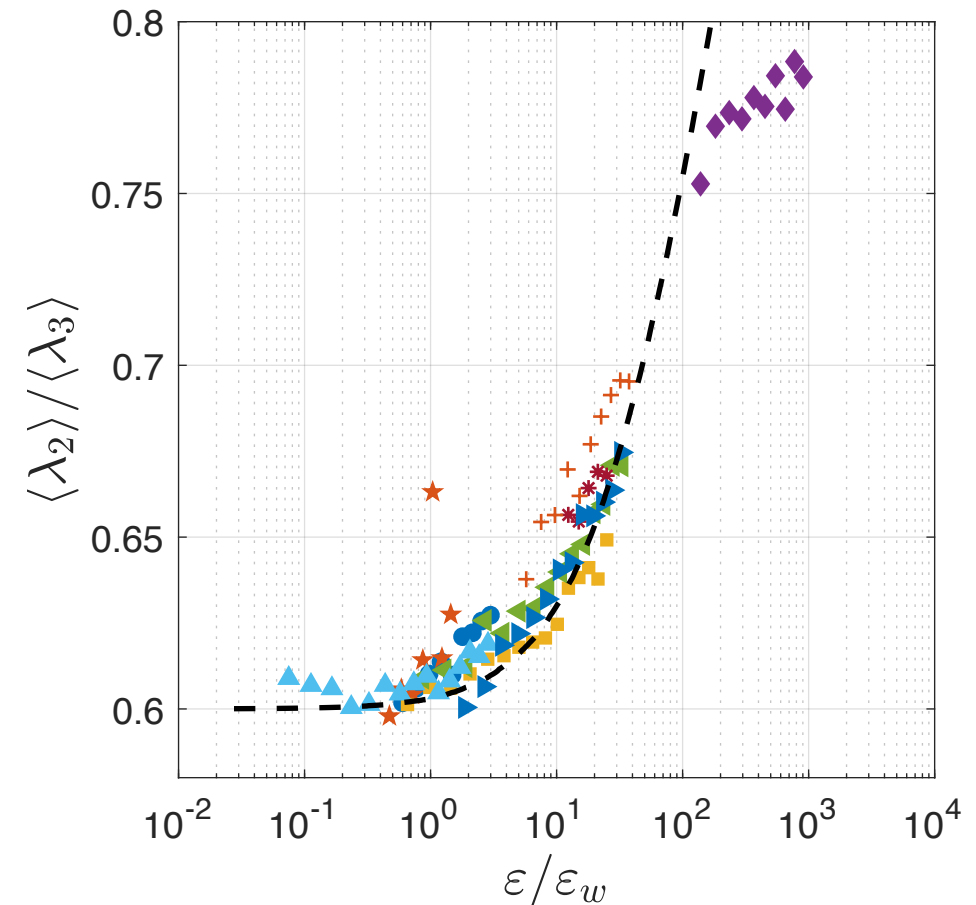
$$L_I \sim 7 \text{ cm}$$

Modeling the threshold of the transition for discs

Main difference with fibers is the timescale of the deformation



λ_i eigenvalues of the moment of the tensor of inertia



$$R_c \sim \frac{B^{3/10}}{(\rho_f \sigma^{1/2} \epsilon)^{1/5}}$$

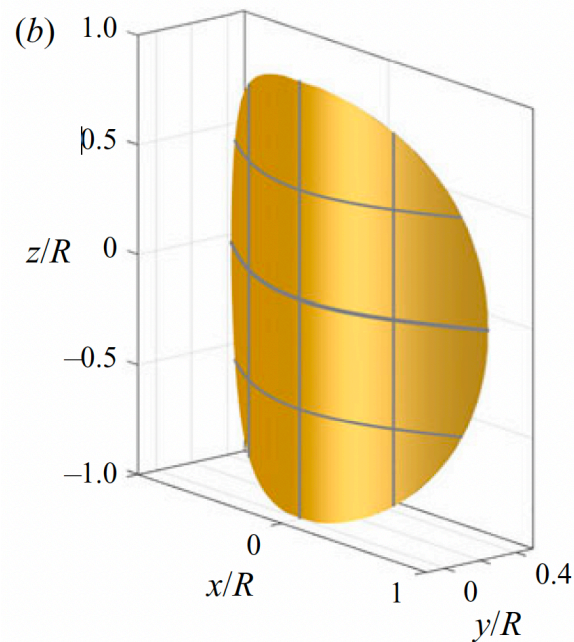
$$E \sim 1 \text{ GPa}$$

$$h \sim 80 \text{ } \mu\text{m}$$

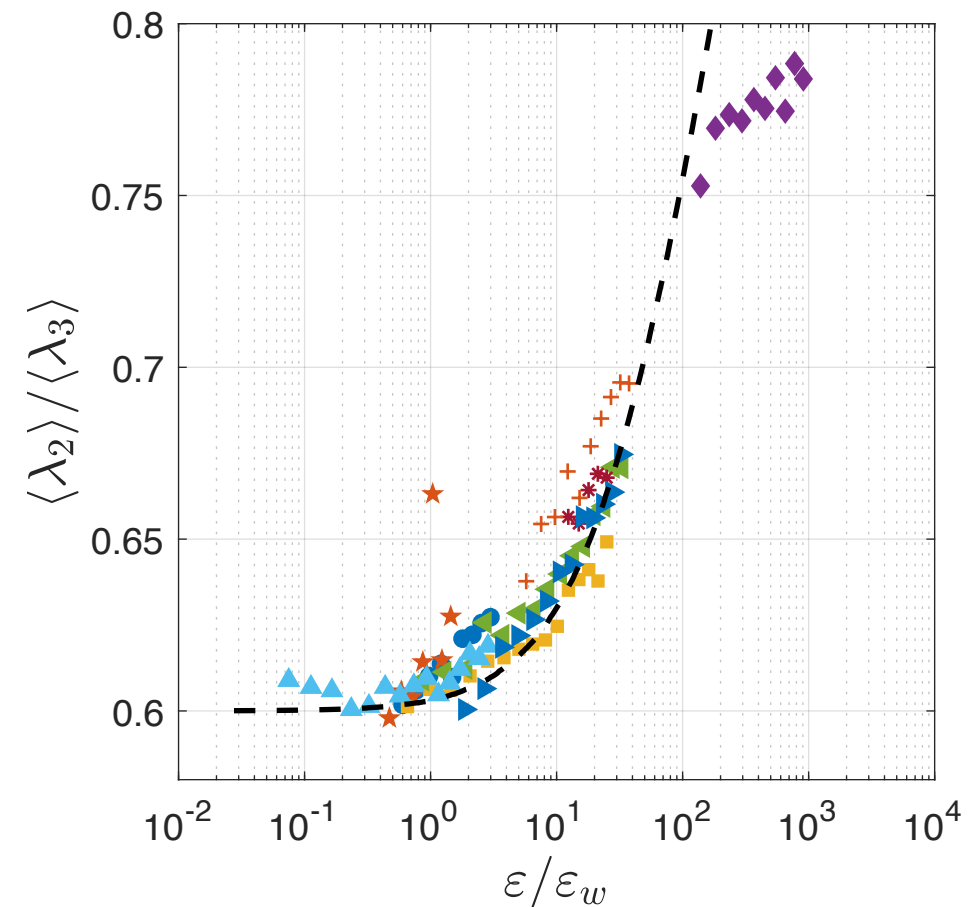
$$R_c \sim 5 - 1 \text{ mm}$$

Modeling the threshold of the transition for discs

Main difference with fibers is the timescale of the deformation



λ_i eigenvalues of the moment of the tensor of inertia



$$R_c \sim \frac{B^{3/10}}{(\rho_f \sigma^{1/2} \epsilon)^{1/5}}$$

$$E \sim 1 \text{ GPa}$$

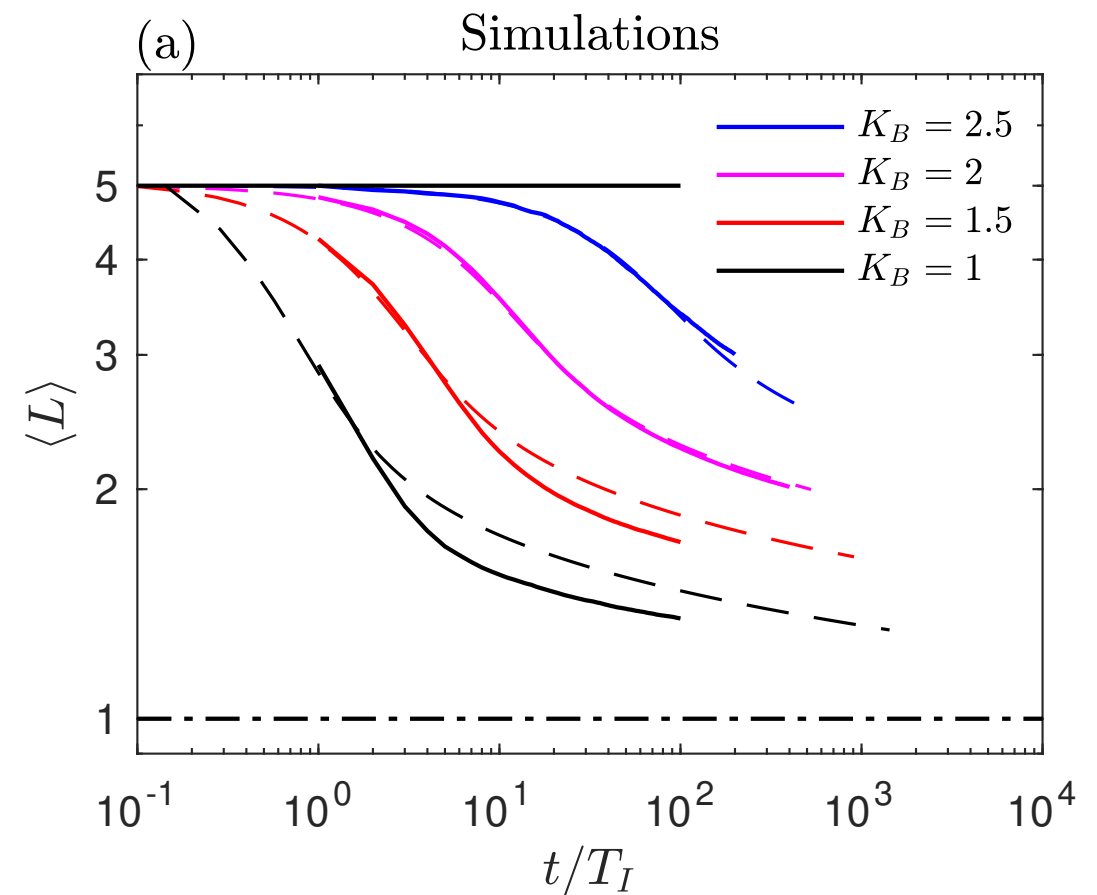
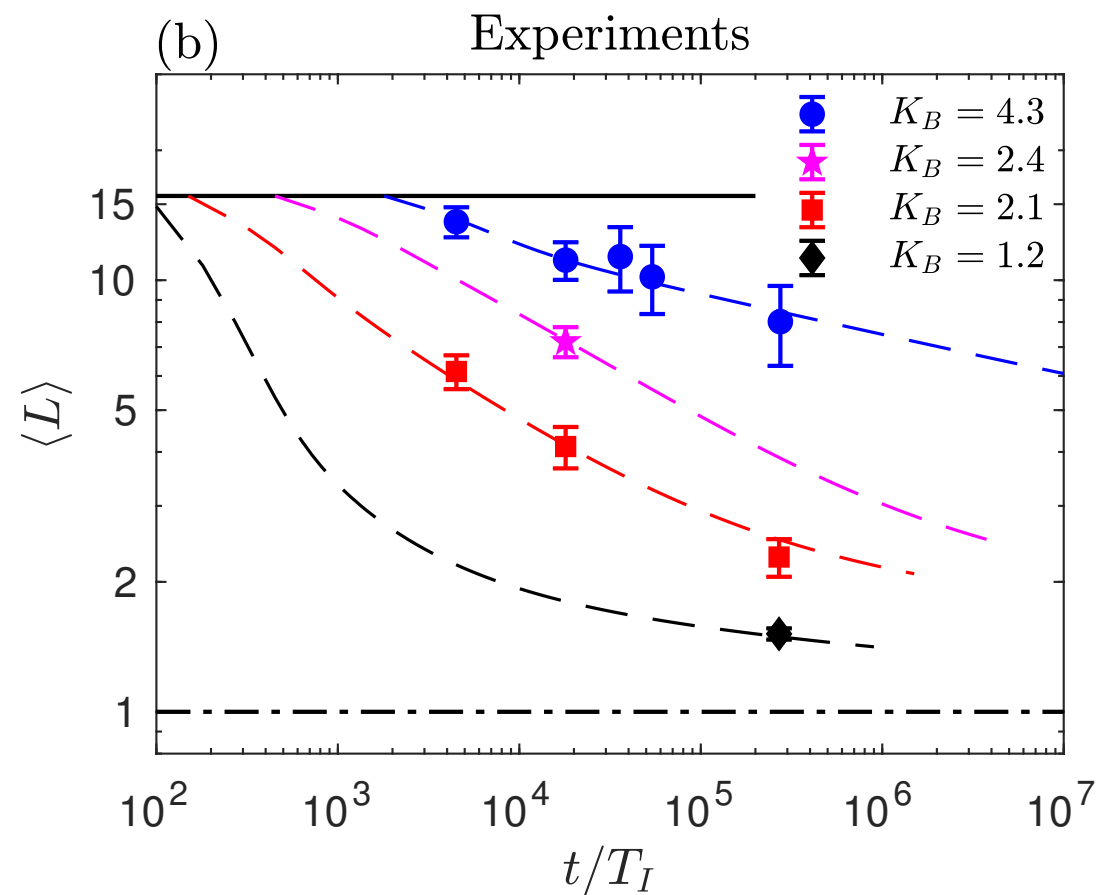
$$h \sim 80 \text{ } \mu\text{m}$$

$$\epsilon \sim 10^{-1} - 10^2 \text{ m}^2 \cdot \text{s}^{-3}$$

$$R_c \sim 5 - 1 \text{ mm}$$

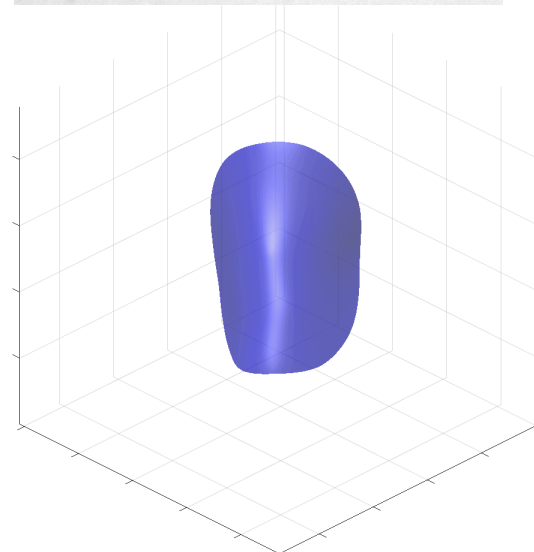
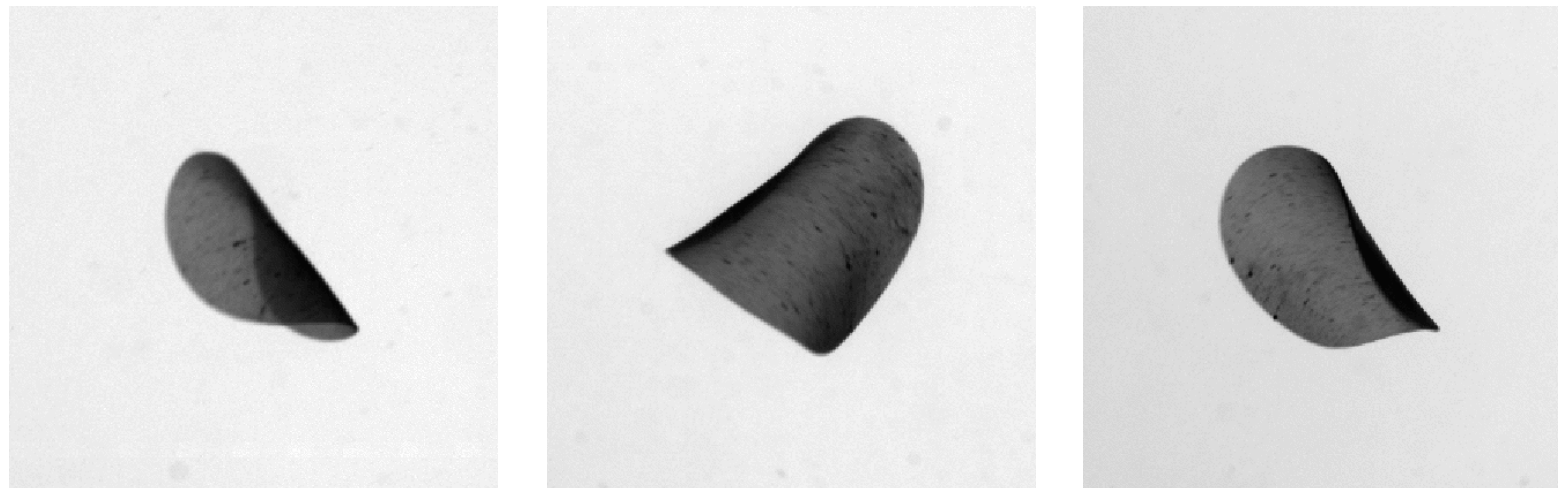
On going and future work

- More complex shapes
- Dynamics of fragmentation



On going and future work

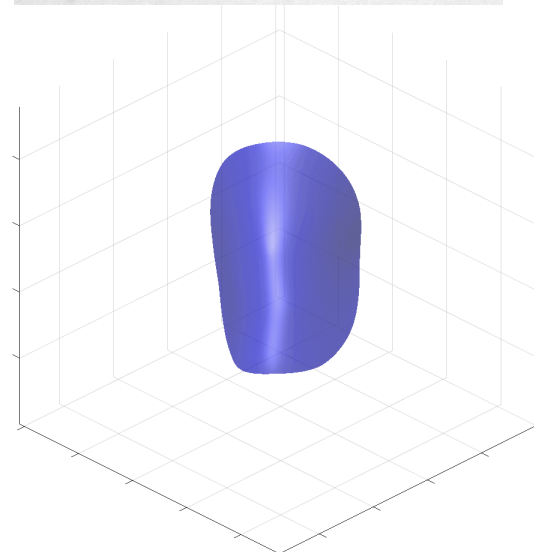
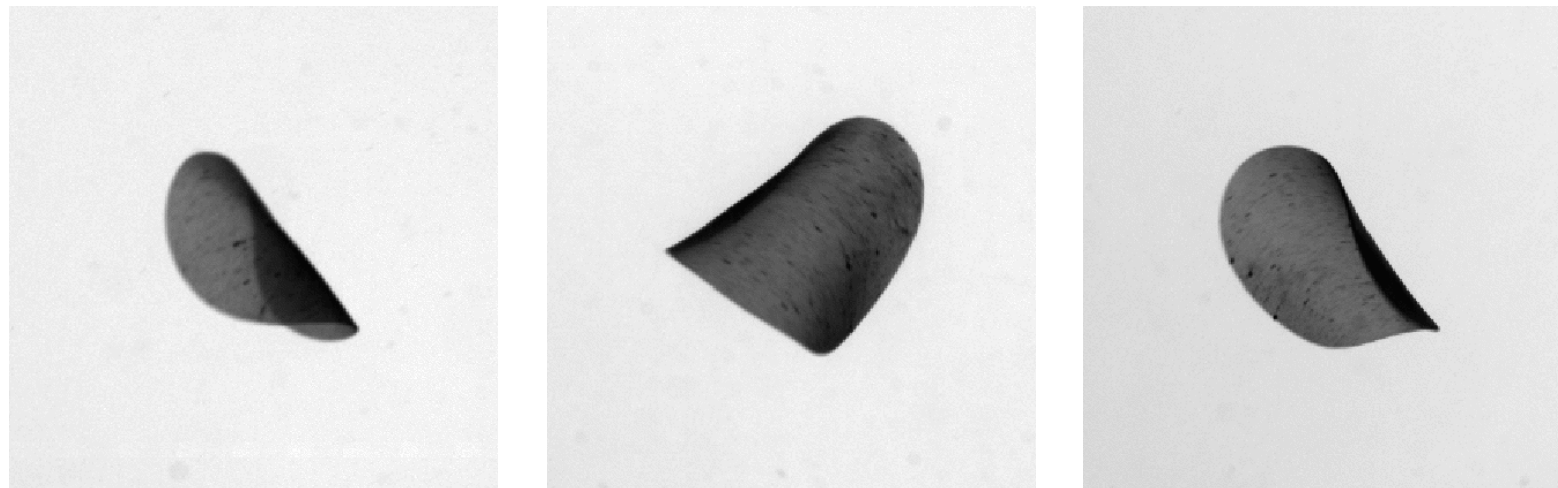
- More complex shape
- Dynamics of fragmentation
- Influence of deformation on the particle dynamics



E. Ibarra

On going and future work

- More complex shape
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Thank you for your attention